MAX PLANCK INSTITUTE
FOR SECURITY AND PRIVACY

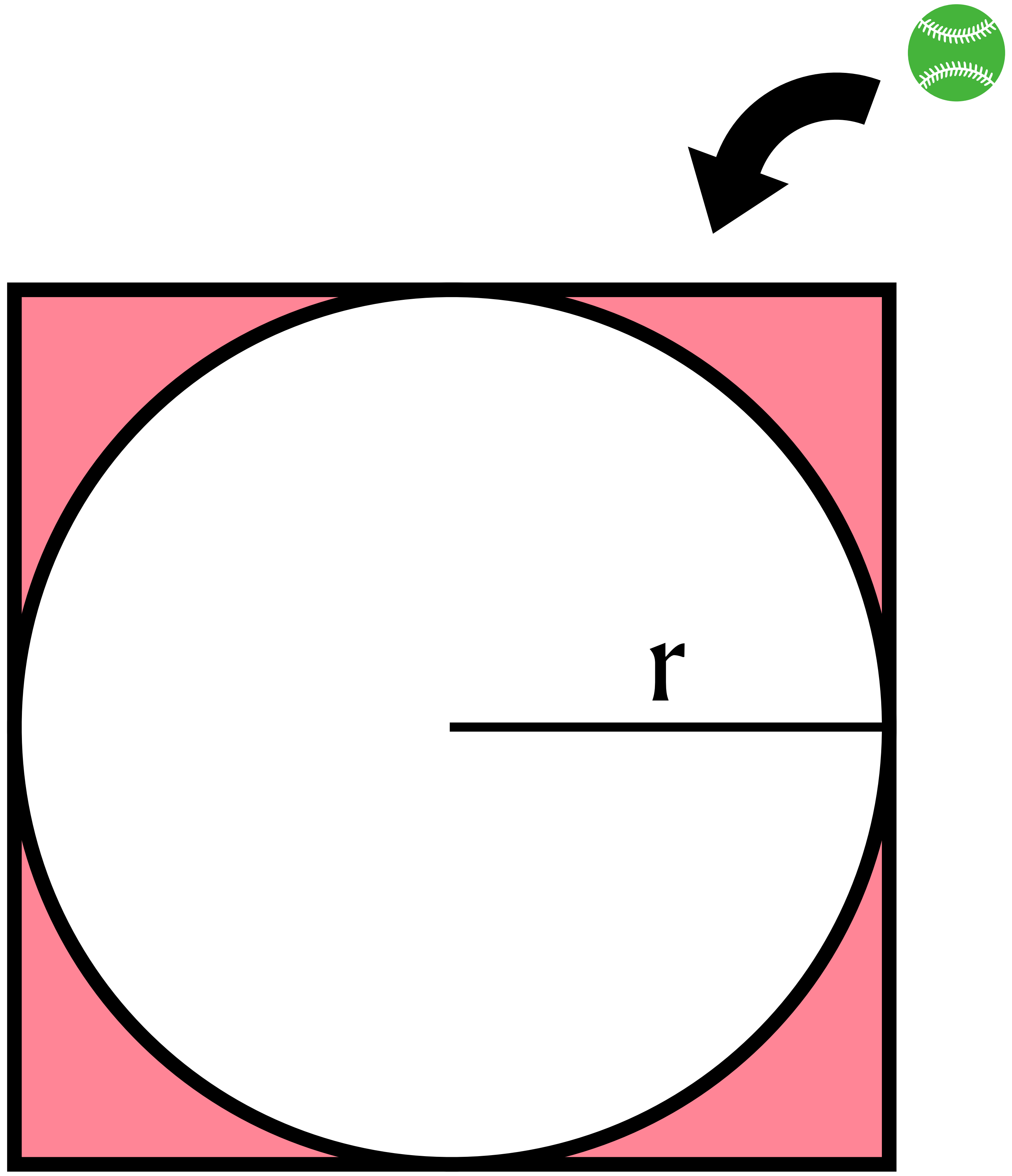
Accounting for Missing Events in Statistical Information Leakage Analysis

Seongmin Lee[1], Shreyas Minocha[2], and Marcel Böhme[1]

1. Max Planck Institute for Security and Privacy (MPI-SP)
2. Georgia Institute of Technology

ICSE 2025

Q. What is the probability of a thrown 🥎 ball to the 🟥 square dropped not into the ? area?



$$P(\neg \text{in white area}) = ?$$

Q. What is the probability of a thrown ball to the square dropped not into the ? area?

Two Ways to Solve the Problem

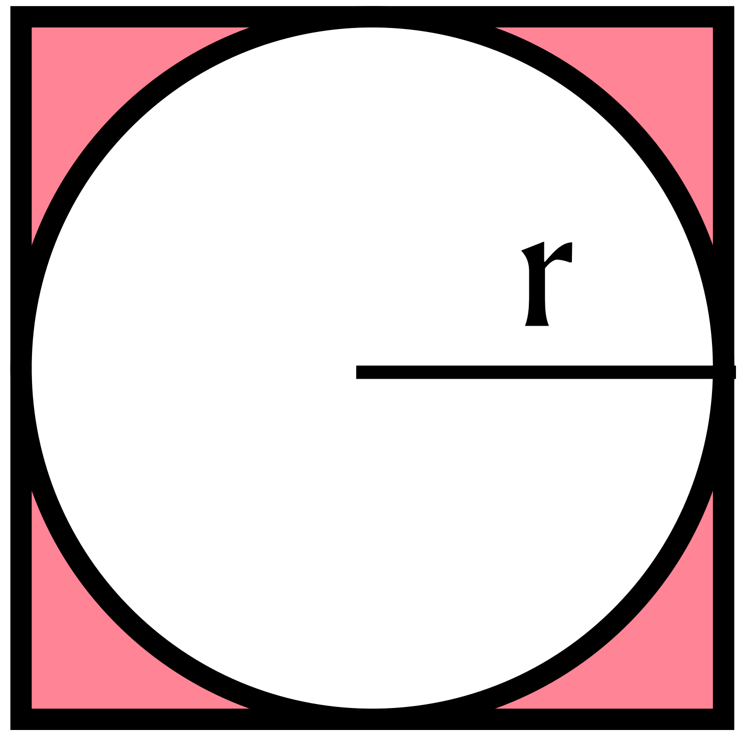
$$P(\neg \text{in white area}) = ?$$

Q. *What is the probability of a thrown ⚾ ball to the 🟥 square dropped not into the ⓠ area?*

1️⃣ Analytic methodology

If the problem can easily be ***mathematically*** *modeled,*

(e.g, area = circle)



$$\Pr(\neg \text{in circle}) = \frac{Area(\text{Square}) - Area(\text{Circle})}{Area(\text{square})} = \frac{(2r)^2 - \pi r^2}{(2r)^2} = \frac{4-\pi}{4} \approx 0.2146...$$

✅ ***Precise result / Formal guarantees***

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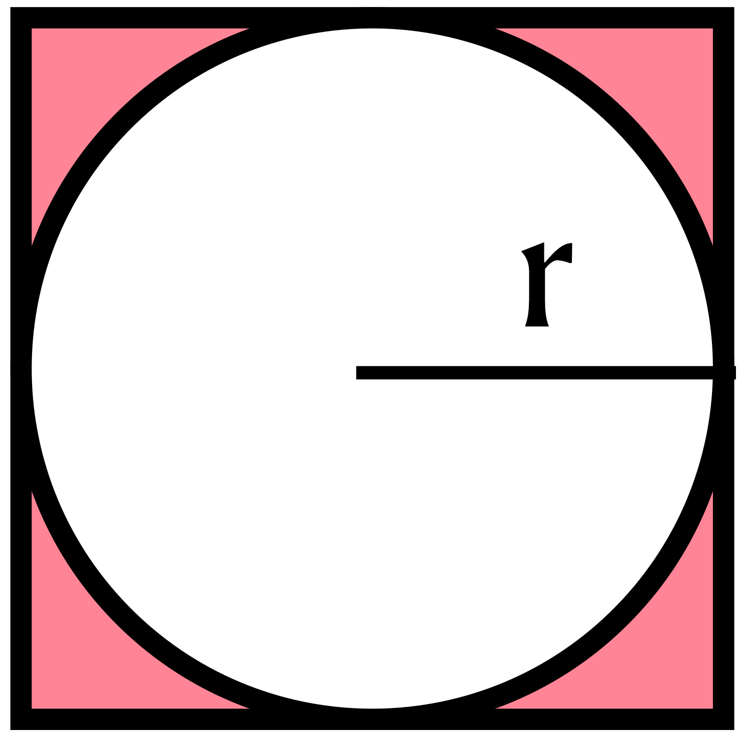

$$
\begin{aligned}
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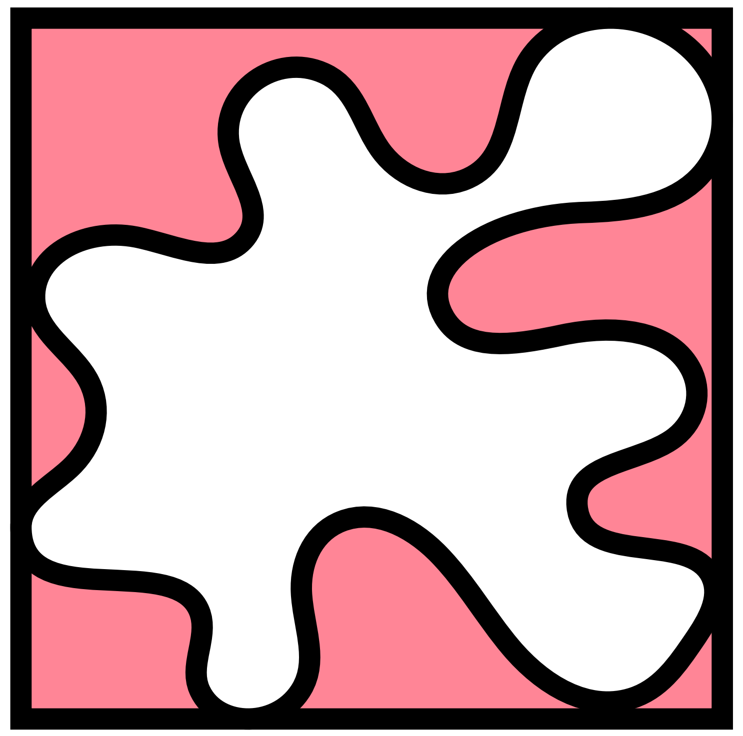

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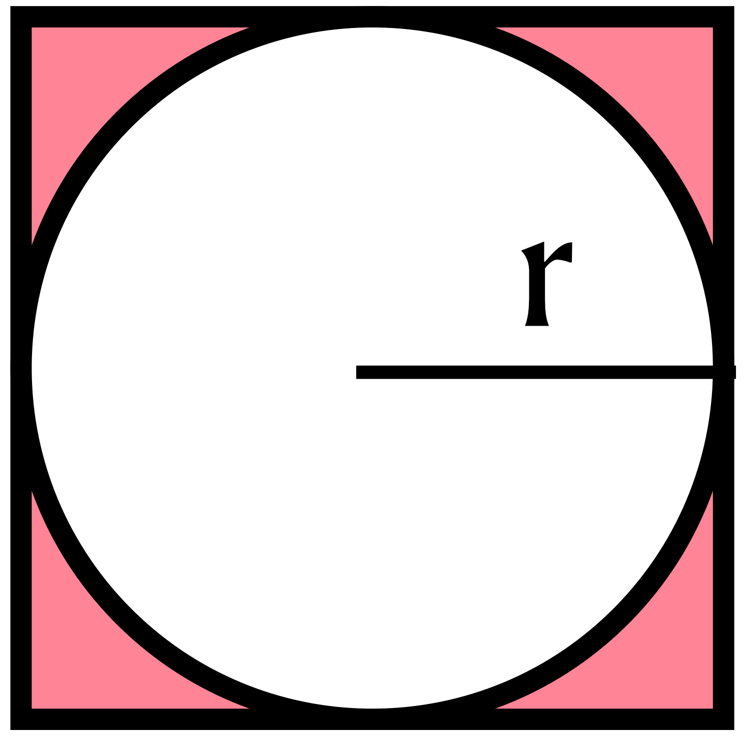

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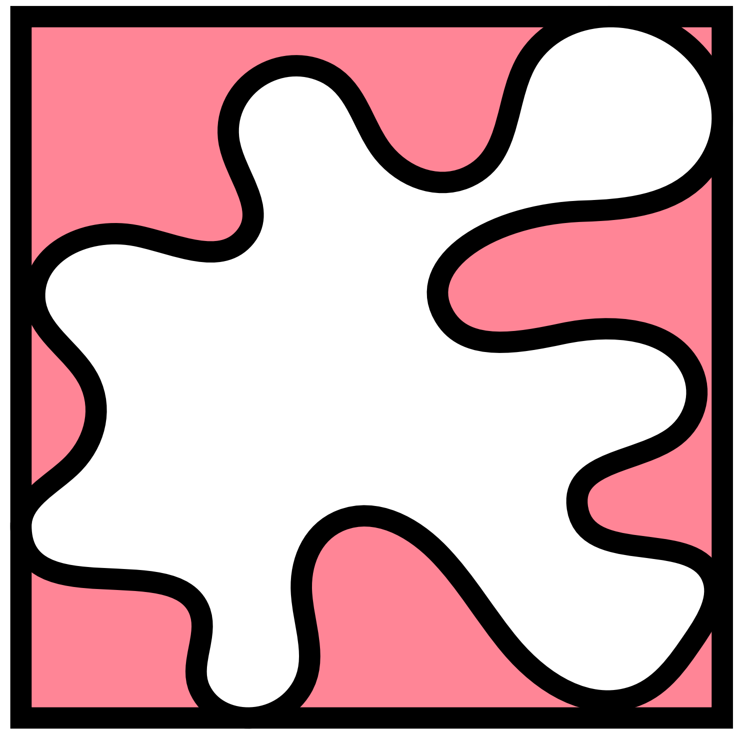

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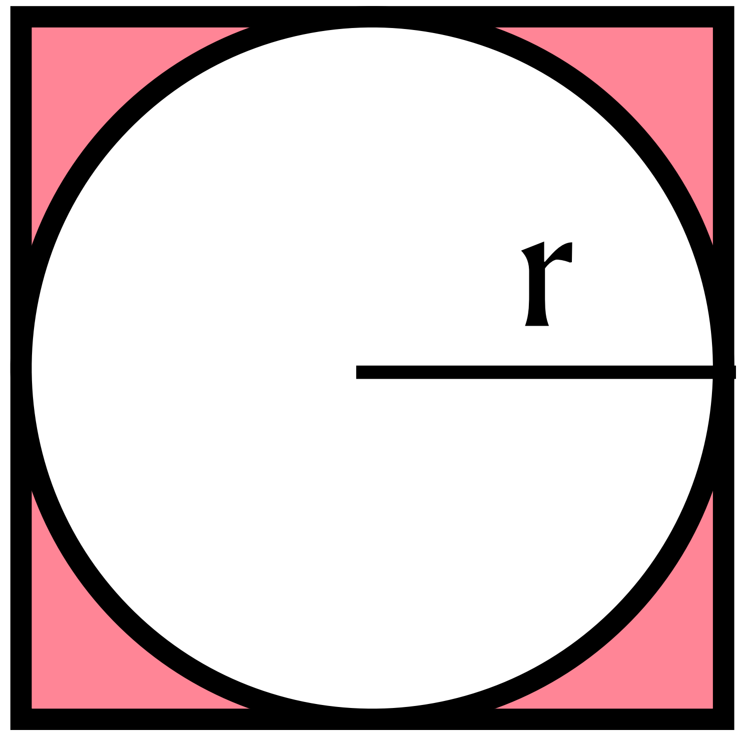

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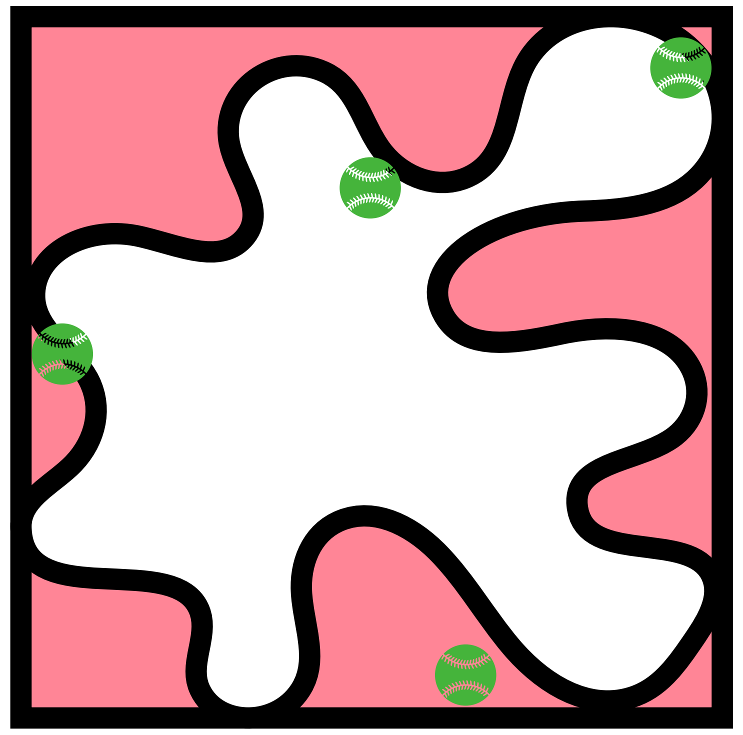
✅ ***Precise result / Formal guarantees***

2️⃣ Empirical methodology

For example, the ***Monte Carlo method****, where we* ***simulate*** *the ball throwing*



$$
\begin{aligned}
&\hat{\Pr}(\neg\text{in area})\\
&= \frac{\#\text{ of balls outside the area}}{\#\text{ of balls thrown}}\\
&= \frac{1}{4} = 0.25
\end{aligned}
$$

Q. What is the probability of a thrown 🥎 ball to the 🟥 square dropped not into the ? area?

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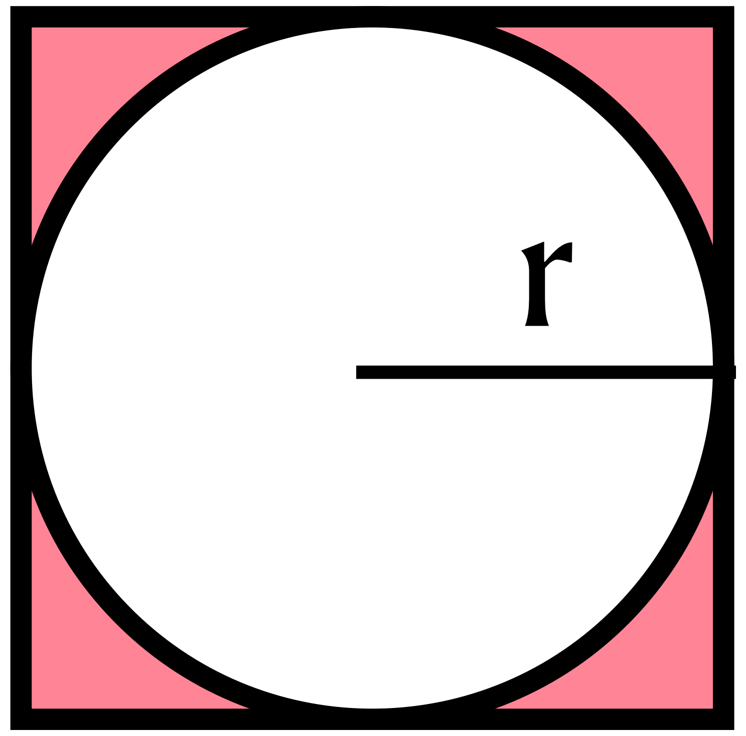

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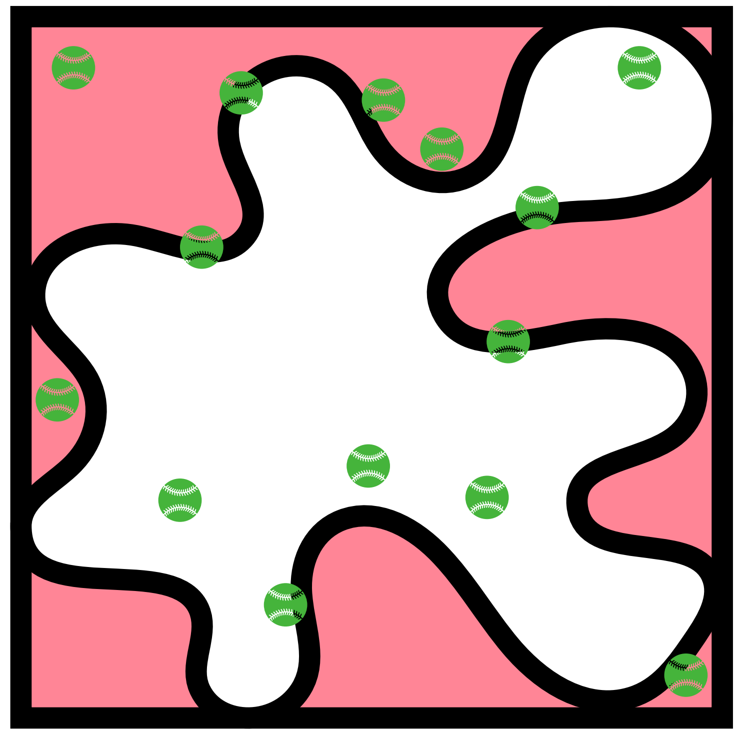

$$\hat{\Pr}(\neg\text{in area}) = \frac{\#\text{ of balls outside the area}}{\#\text{ of balls thrown}} = \frac{5}{14} \approx 0.3571$$

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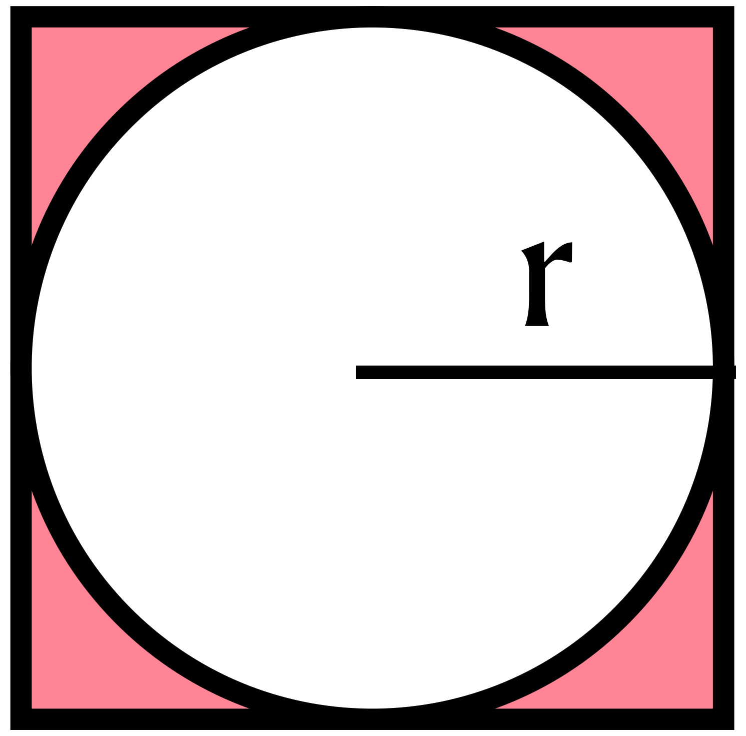

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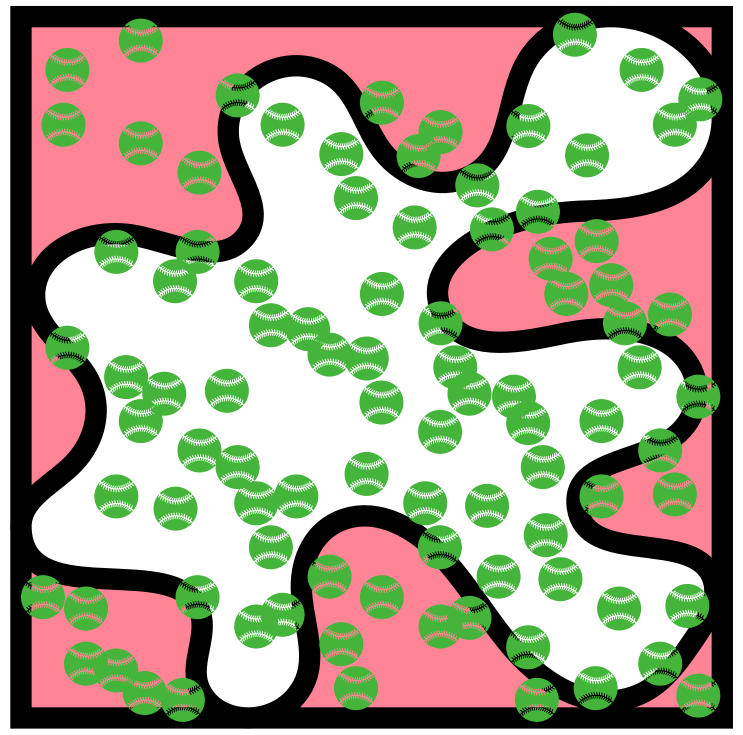

$$\hat{\Pr}(\neg\text{in area}) = \frac{\#\text{ of balls outside the area}}{\#\text{ of balls thrown}} = \frac{3577}{10000} = 0.3577$$

Q. *What is the probability of a thrown 🎾 ball to the 🟥 square dropped not into the ? area?*

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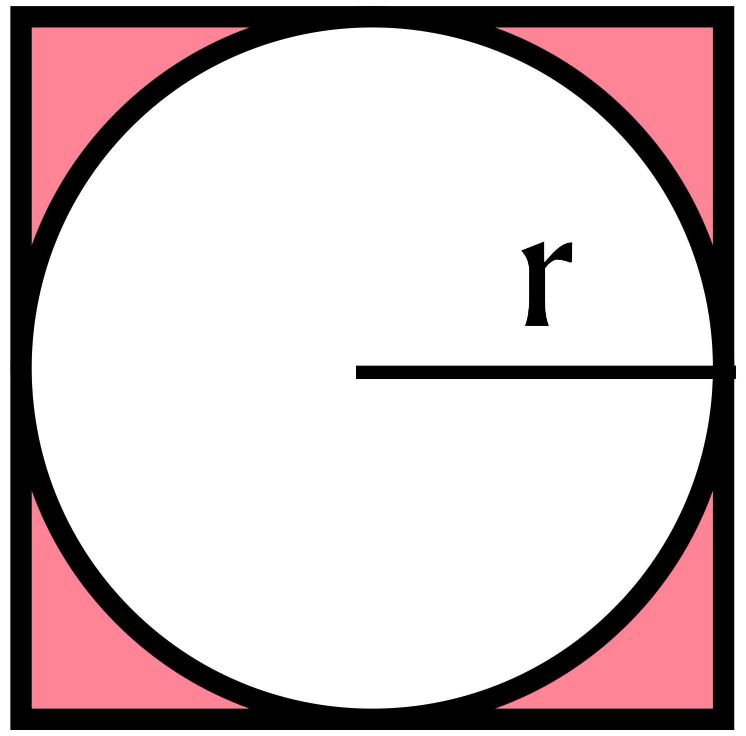

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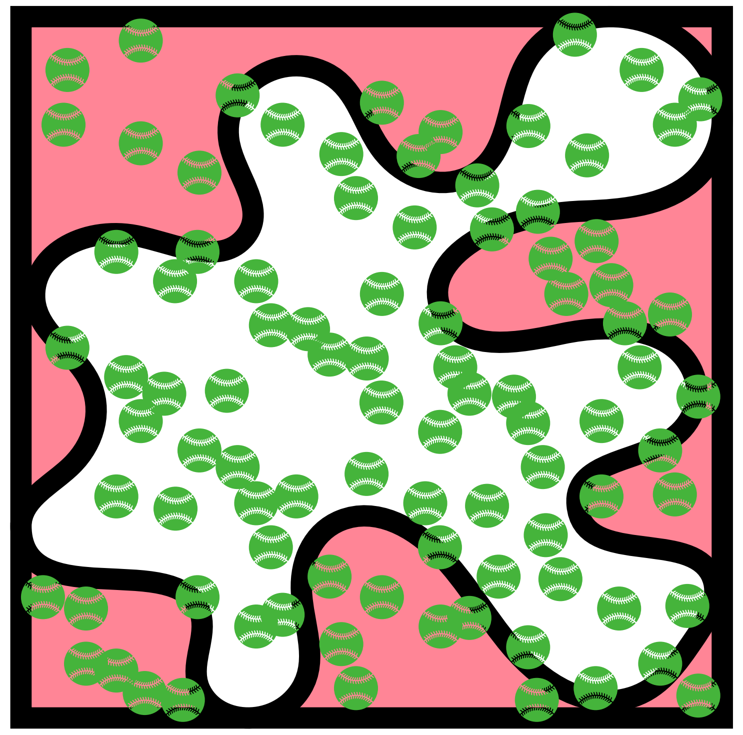

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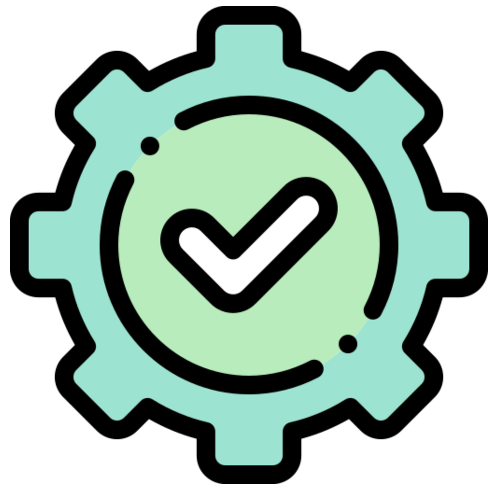
✅ ***Scalable,** i.e., can deal with complex problems*

Information Leakage Analysis

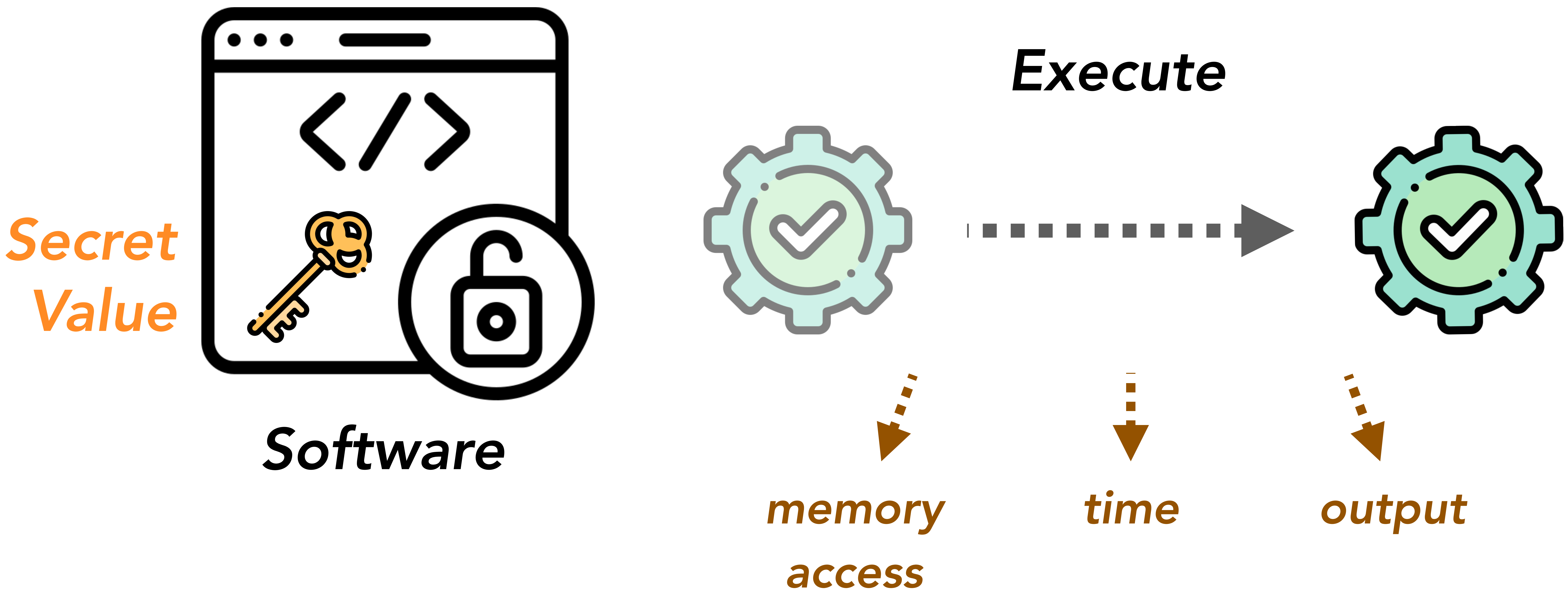
Information Leakage

Secret Value

Software



Information Leakage



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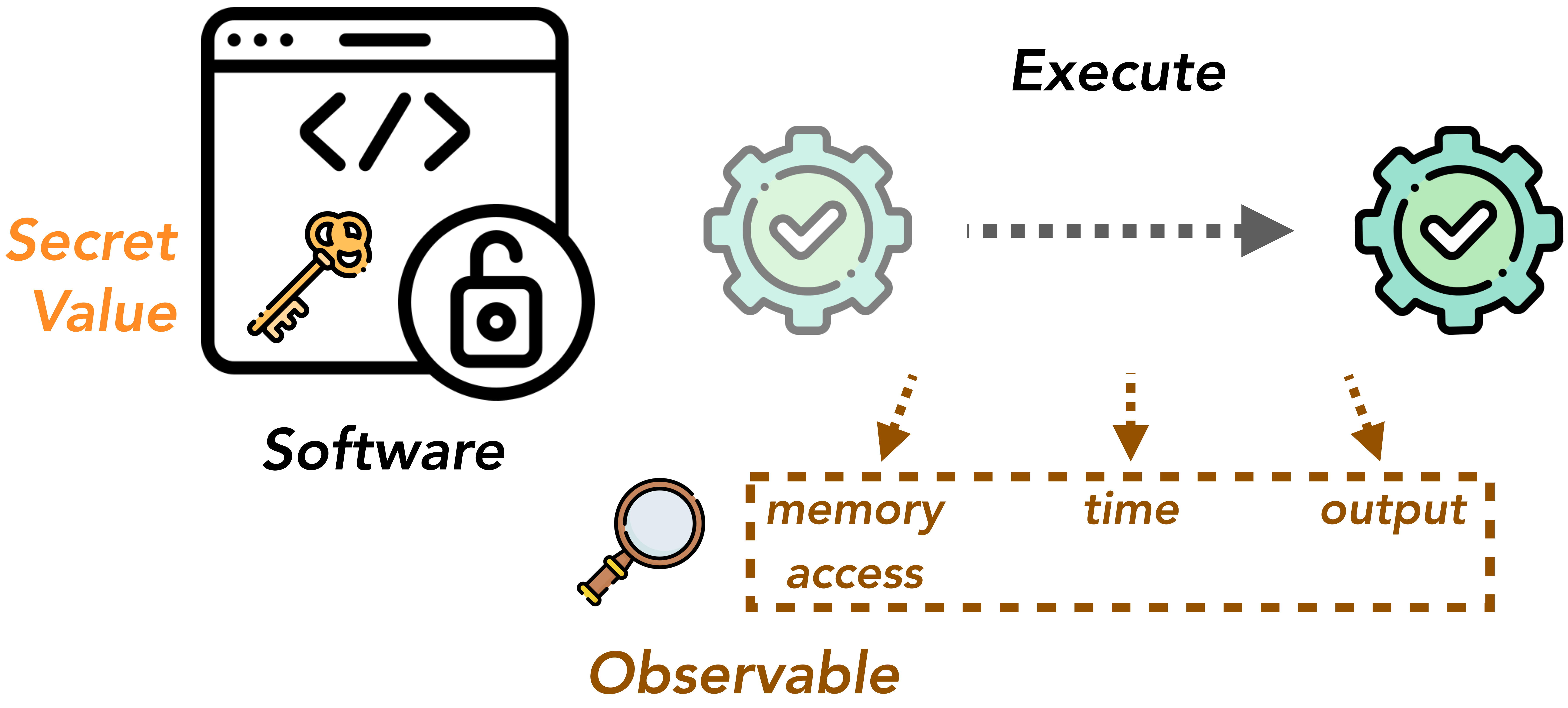

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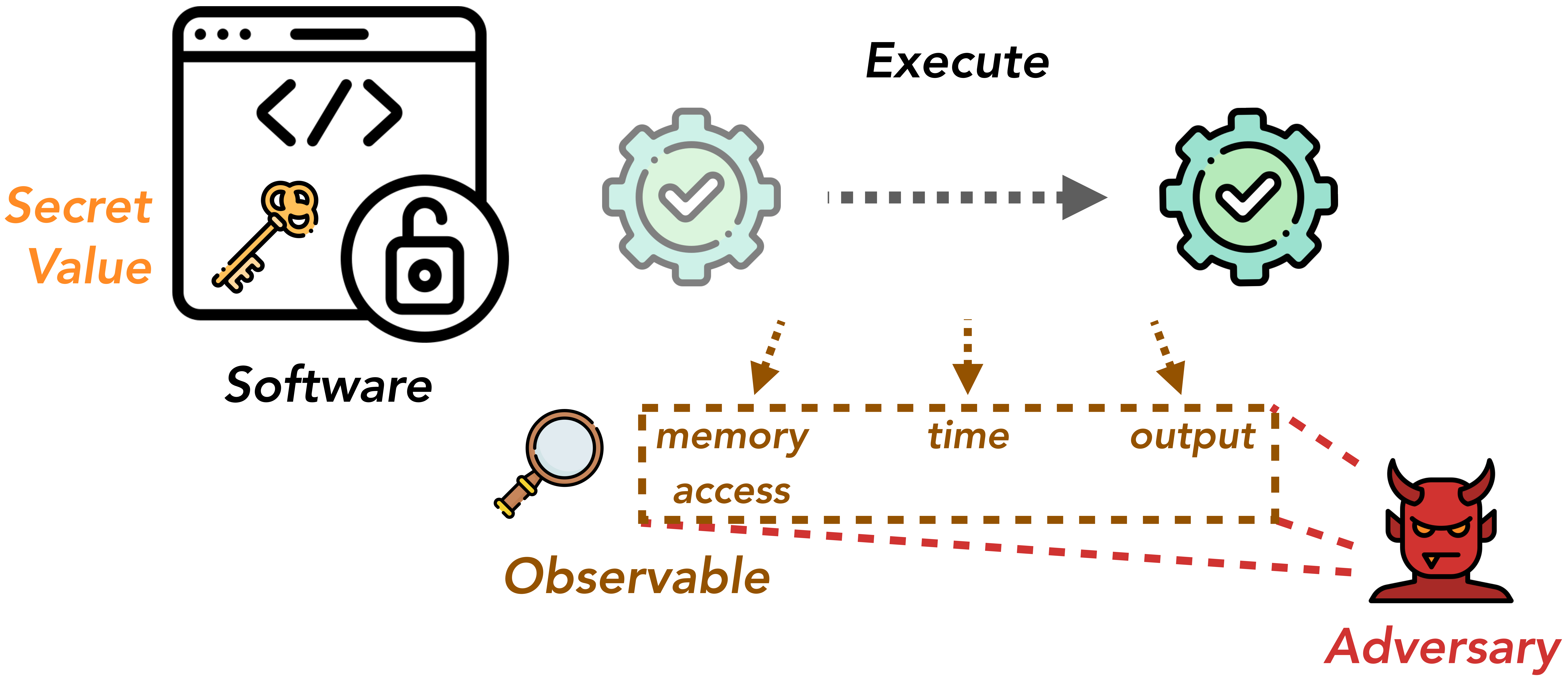
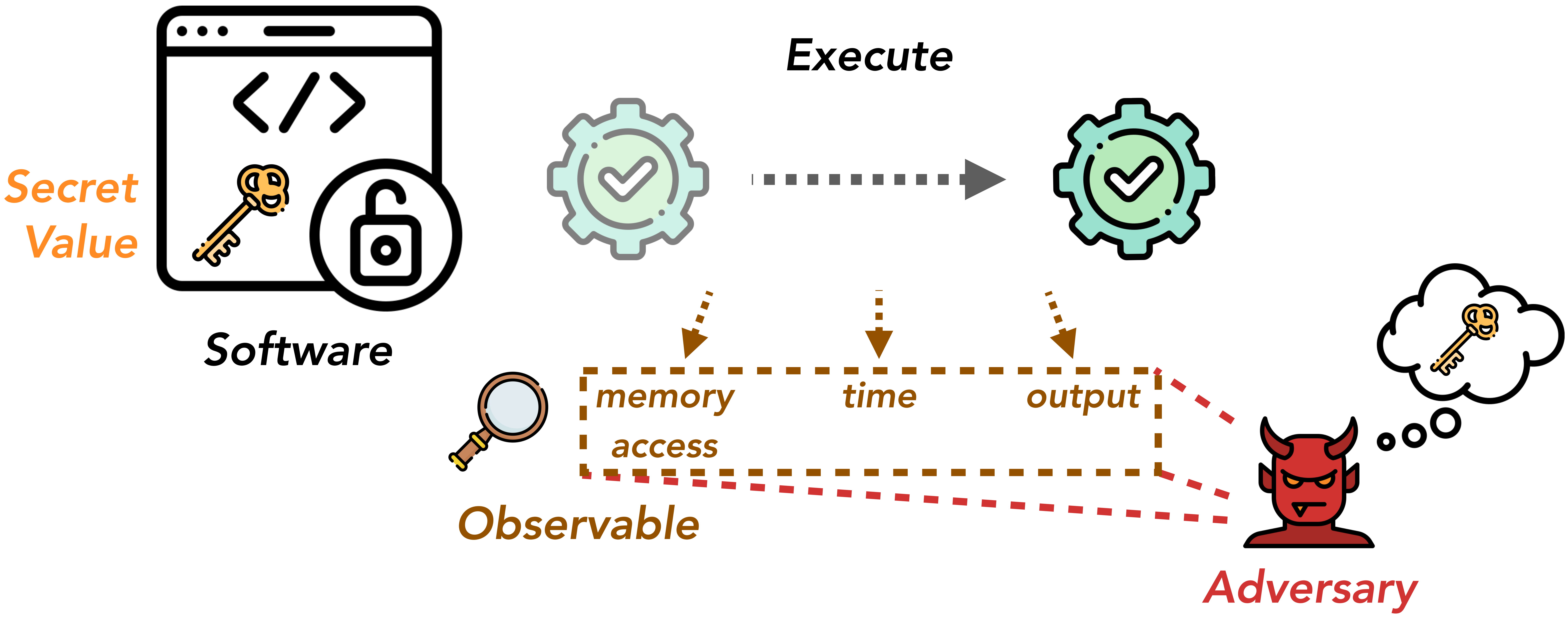


Information Leakage



Measure of Information Leakage

- The amount of information about the secret (S) was leaked from the observable (O):

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 $(S$  $)$

Initial uncertainty of the ***secret value***

[Notations]

 : *Secret*

 : *Uncertainty*

 : *Observable*

Measure of Information Leakage

- The amount of information about the secret (S) was leaked from the observable (O):

$$?(S) - ?(S \mid O)$$

Initial uncertainty of the ***secret value***

Remaining uncertainty of the ***secret value*** *after checking the* ***observable value***

[Notations]

 : Secret

 : Uncertainty

 : Observable

Measure of Information Leakage

- The amount of information about the secret (S) was leaked from the observable (O): $?(S) - ?(S \mid O)$
- The ***Uncertainty*** ? can be measured with **Shannon Entropy** H.
  - *If the distribution D's entropy H(D) is X, it means ~2^X times of guessing are expected to match a sample from D.*

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$$H(S) = -\sum_{s \in S} \Pr_S(s) \cdot \log_2 \Pr_S(s)$$

*Initial uncertainty of the **secret value***

marginal prob. dist. of secret S

[Notations]

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: Uncertainty

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- The amount of information about the secret (S) was leaked from the observable (O): $?(S) - ?(S \mid O)$
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$$H(S \mid O)$$

*Remaining uncertainty of the **secret value** after checking the **observable value***

$$= - \sum_{(s,o) \in S \times O} \Pr(s, o) \cdot \log_2 \frac{\Pr(s, o)}{\Pr_O(o)}$$

marginal prob. dist. of observable O

[Notations]

: Secret

: Uncertainty

: Observable

Measure of Information Leakage

Observable (O)

Secret (S)

			...	
	0.085	3E-04	...	0.025
	0.002	0.078	...	1E-04
...	...	...	...	...
	0.012	0.042	...	0.040

Joint Probability Distribution

Measure of Information Leakage

$$H(S)$$

*Initial uncertainty of the **secret value***

Observable (O)

Secret (S)

			...	
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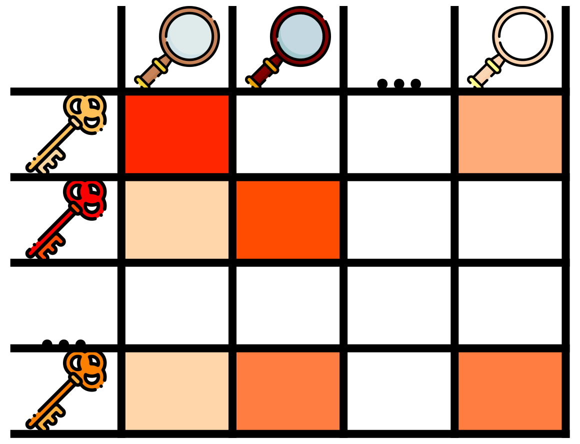
Joint Probability Distribution

$H(\ \mid O\ )$

*Remaining uncertainty of the **secret value** after checking the **observable value***

Measure of Information Leakage

Mutual Information (MI) I measures the information leakage from  .

$$I(S\,;O\,) \quad = \quad H(S\,) - H(\;\mid O\,)$$

Mutual Information (MI)

*between the **secret** and the **observable***

*Initial uncertainty of the **secret value***

*Remaining uncertainty of the **secret value** after checking the **observable value***

Software

Obtaining Information Leakage Bounds via Approximate Model Counting*

SEEMANTA SAHA†, UC Santa Barbara, USA
SURENDRA GHENTIYALA†, UC Santa Barbara, USA
SHIHUA LU, UC Santa Barbara, USA
LUCAS BANG, Harvey Mudd College, USA
TEVFIK BULTAN, UC Santa Barbara, USA

Information leaks are a significant problem in modern software systems. In recent years, information theoretic concepts, such as Shannon entropy, have been applied to quantifying information leaks in programs. One recent approach is to use symbolic execution together with model counting constraints solvers in order to quantify information leakage. There are at least two reasons for unsoundness in quantifying information leakage using this approach: 1) Symbolic execution may not be able to explore all execution paths, 2) Model counting constraints solvers may not be able to provide an exact count. We present a sound symbolic quantitative information flow analysis that bounds the information leakage both for the cases where the program behavior is not fully explored and the model counting constraint solver is unable to provide a precise model count but provides an upper and a lower bound. We implemented our approach as an extension to KLEE for computing sound bounds for information leakage in C programs.

CCS Concepts: • **Software and its engineering → Formal software verification**; **General programming languages**.

Additional Key Words and Phrases: Quantitative Program Analysis, Symbolic Quantitative Information Flow Analysis, Model Counting, Information Leakage, Optimization

ACM Reference Format:
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1 INTRODUCTION

One of the most critical security issues in software systems today is protecting users' private information, which makes analyzing information leakage in software systems a timely and important research problem. A classic approach to address this problem is enforcing *noninterference* which ensures that publicly observable properties of program execution (such as public outputs or side-channels) are independent of secret input values. But, enforcing noninterference is often not possible as software systems need to reveal some amount of information that depends on secret inputs. Consider a password checker where, as public output, the system needs to provide

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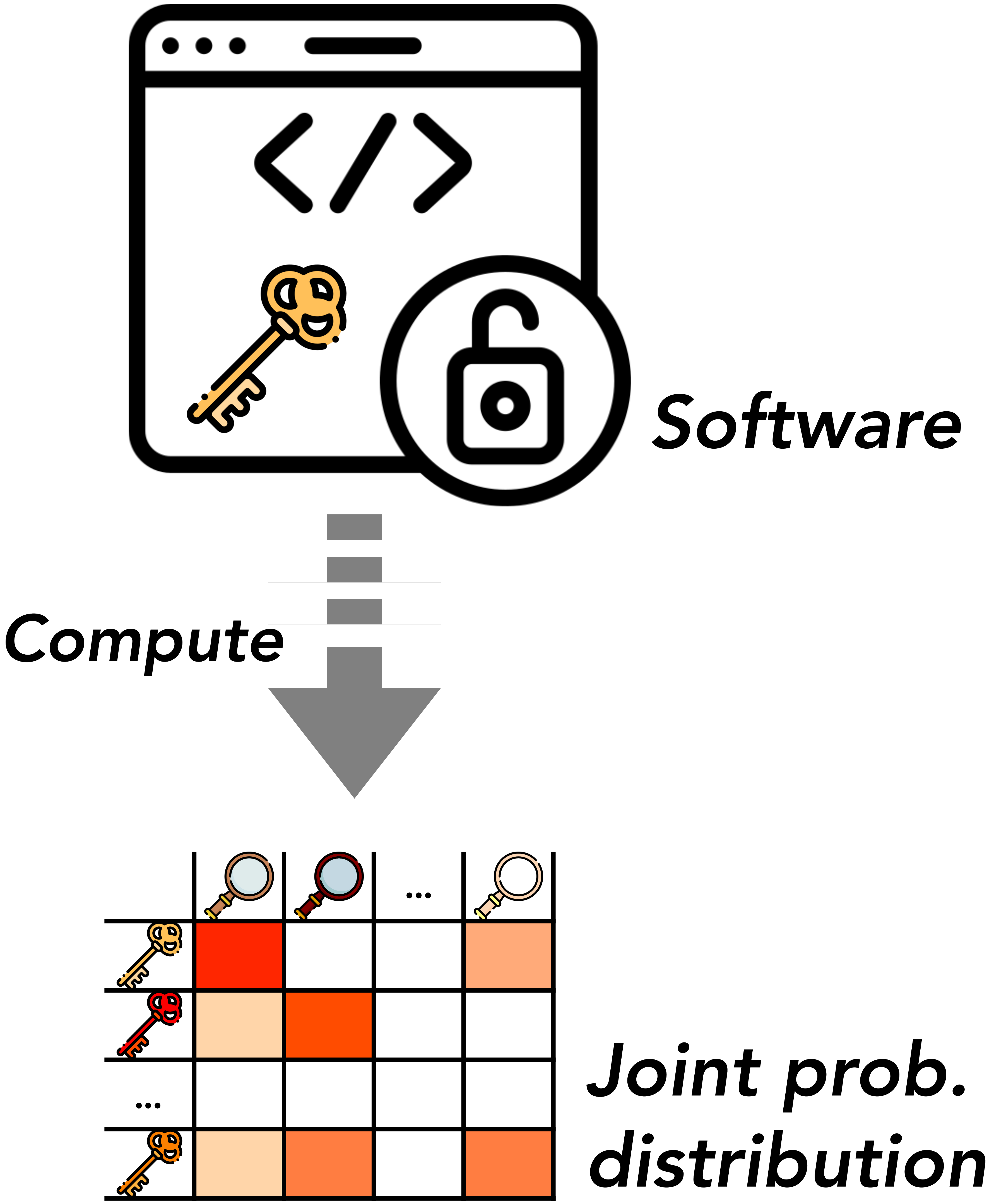
Analytic approach

Uses model counting



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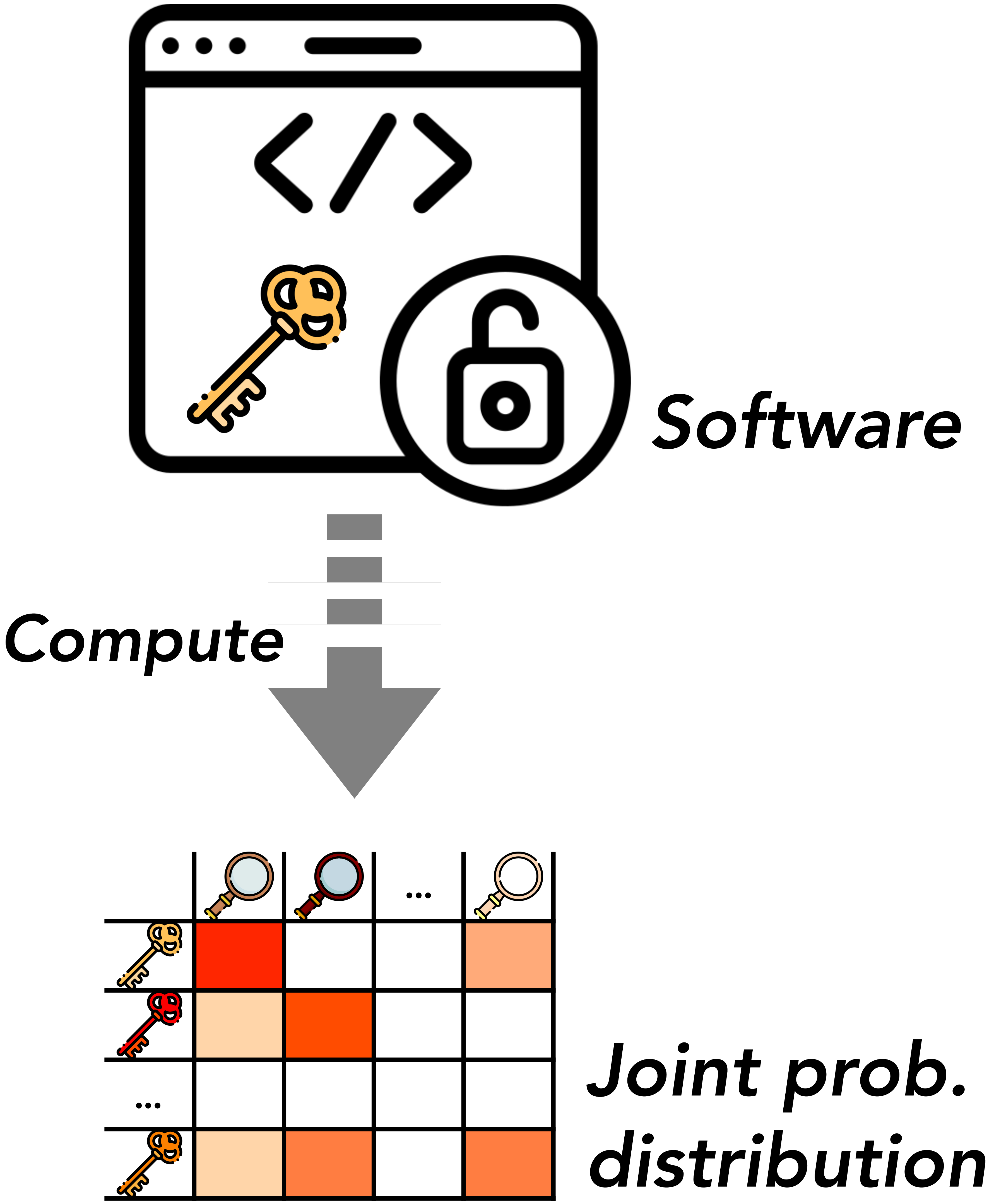
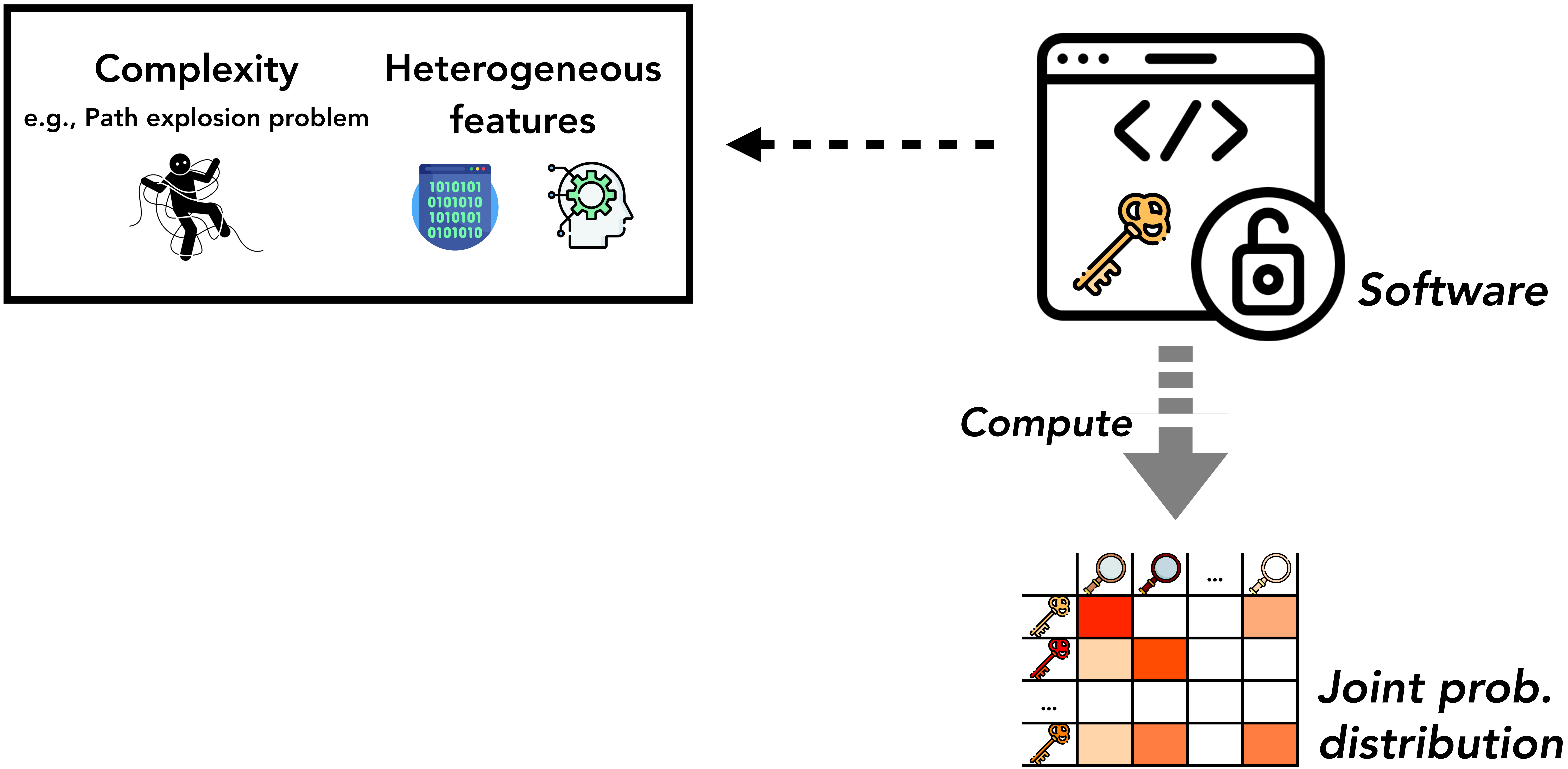
Analytic approach

Uses model counting

*An **analytic approach** provides*
*a **precise result** or*
*a **formal guarantee!***

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e.g., Path explosion problem
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1010101
0101010
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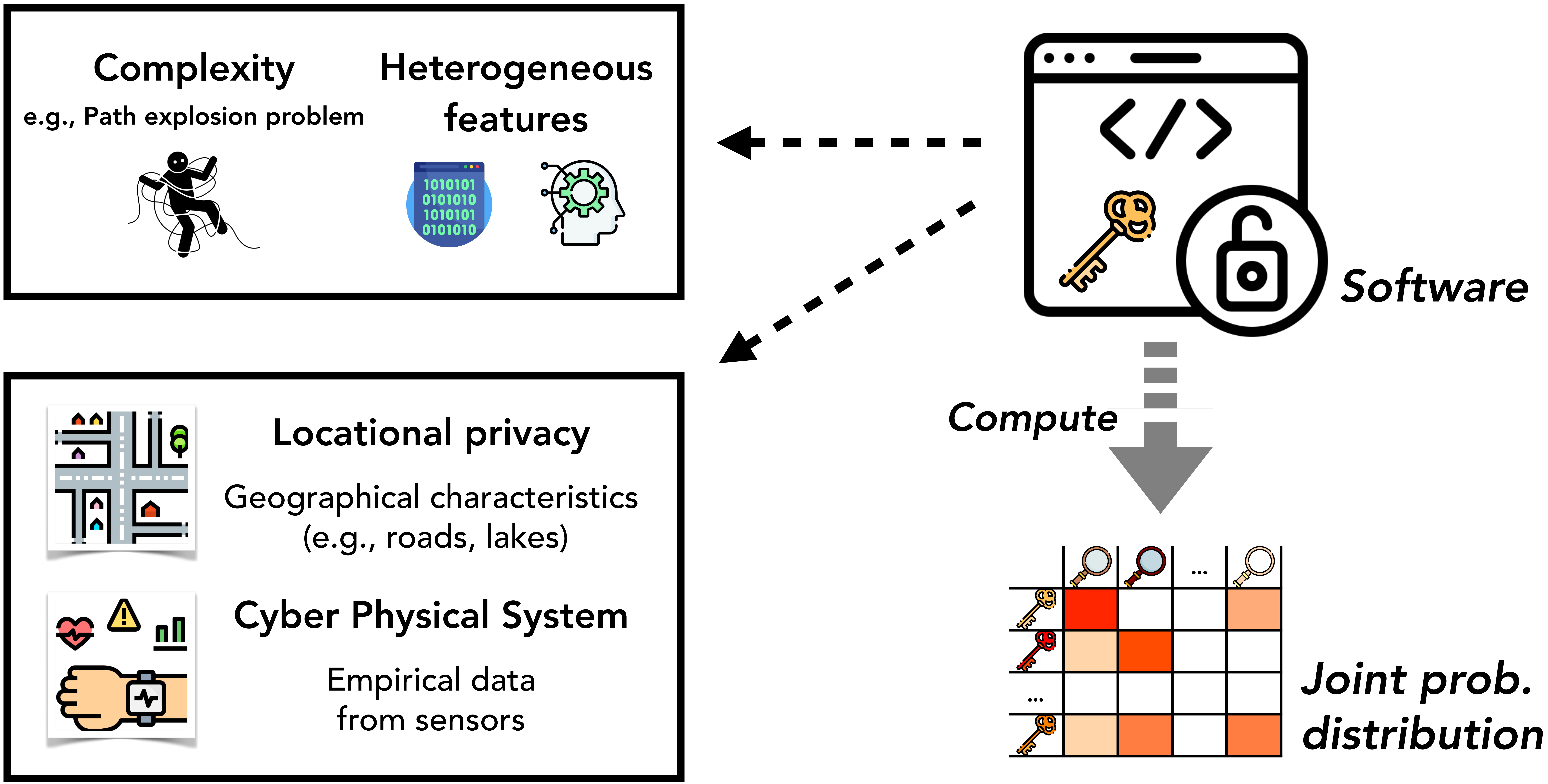
Compute
Joint prob. distribution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Geographical characteristics
(e.g., roads, lakes)
Cyber Physical System
Empirical data
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Software
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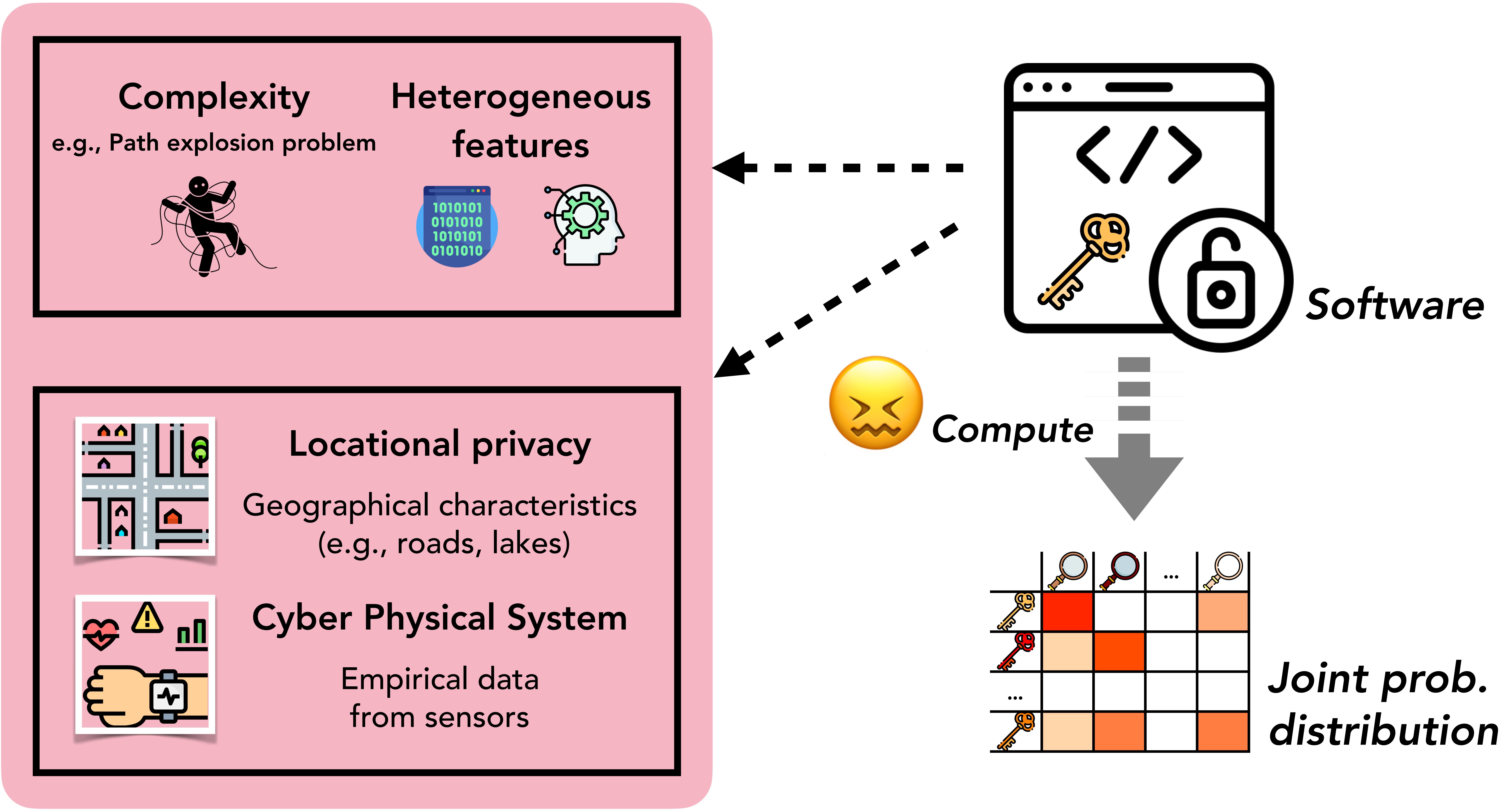
...
Joint prob.
distribution



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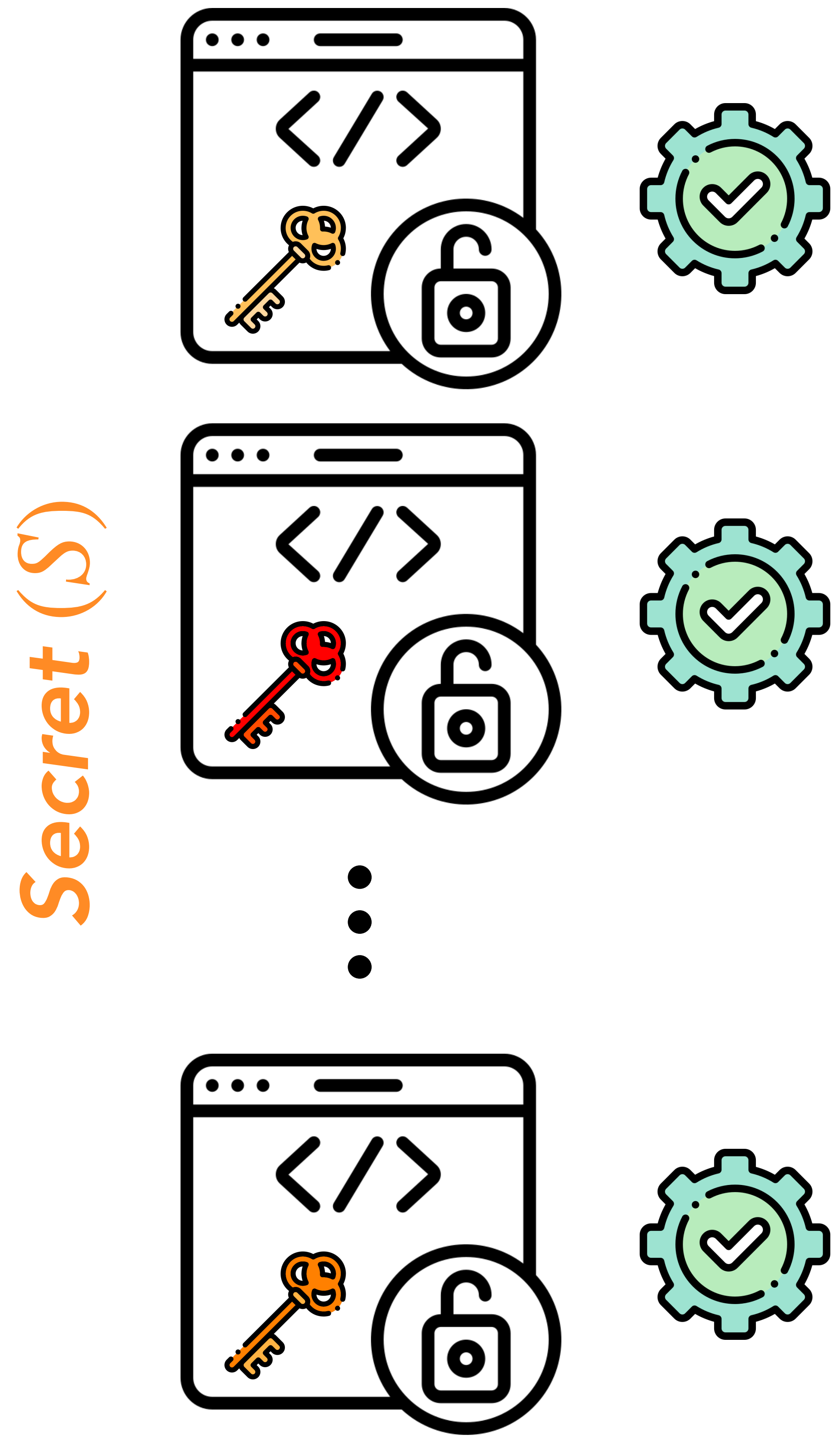
(Existing) Empirical Information Leakage Analysis



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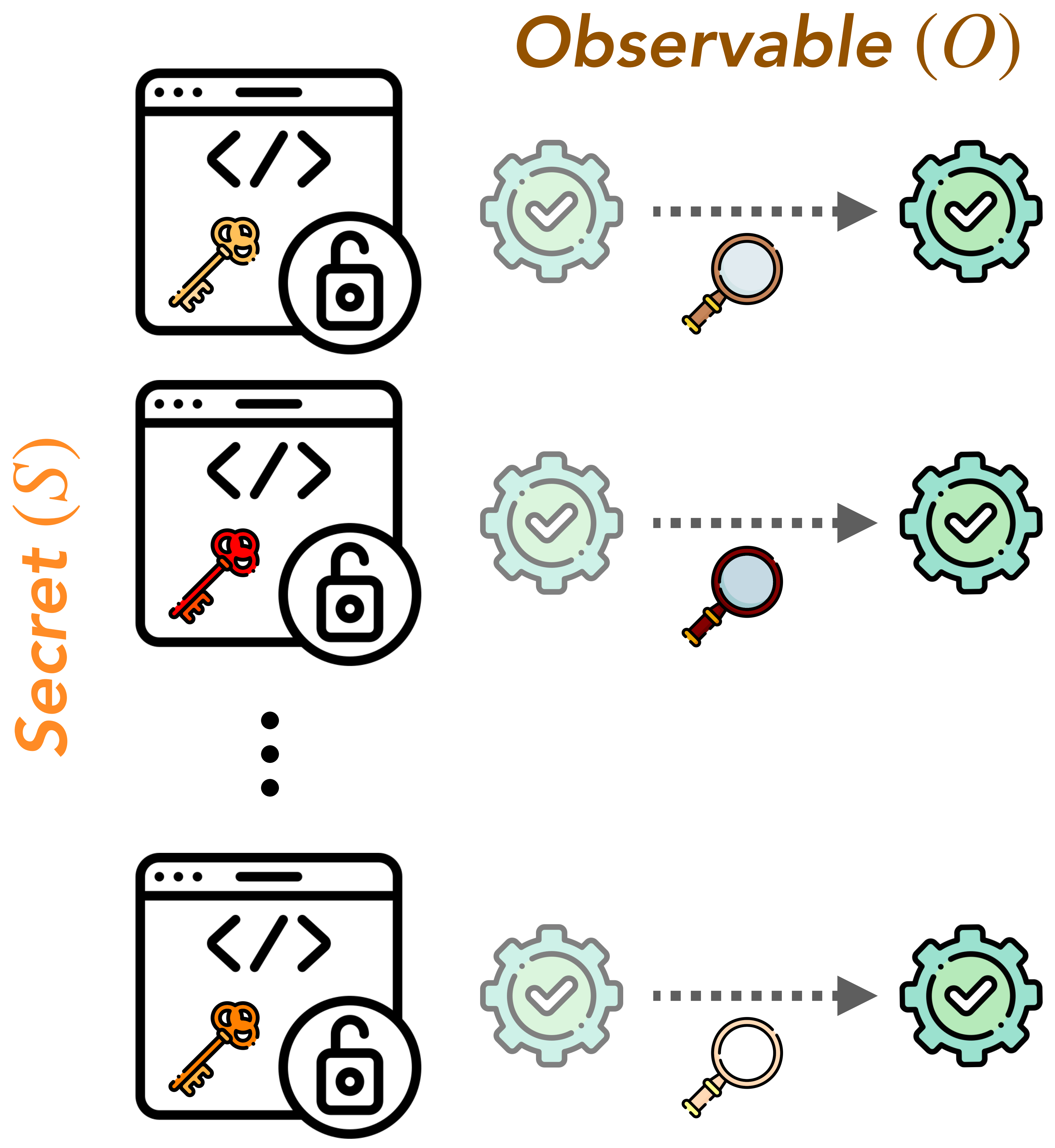

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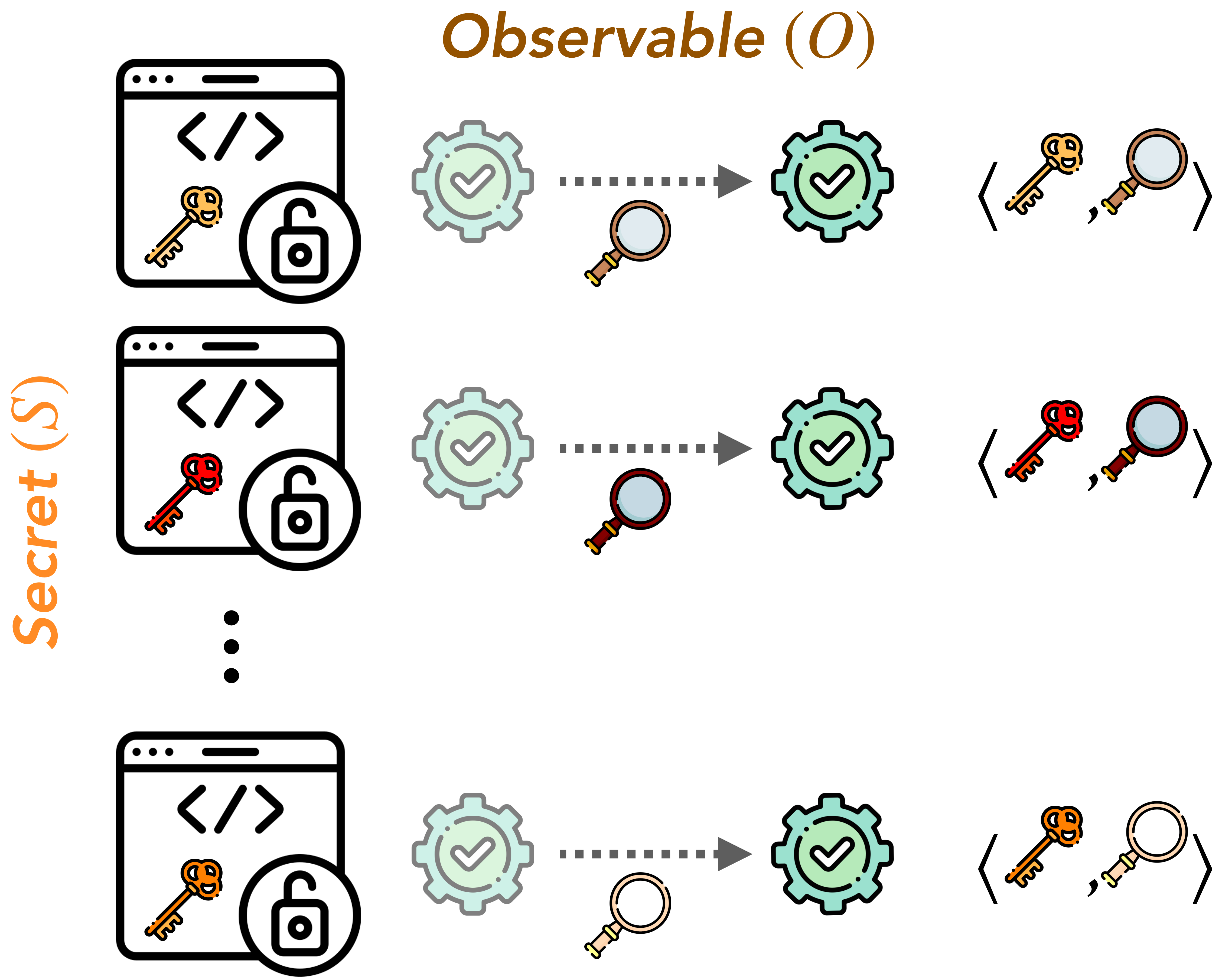

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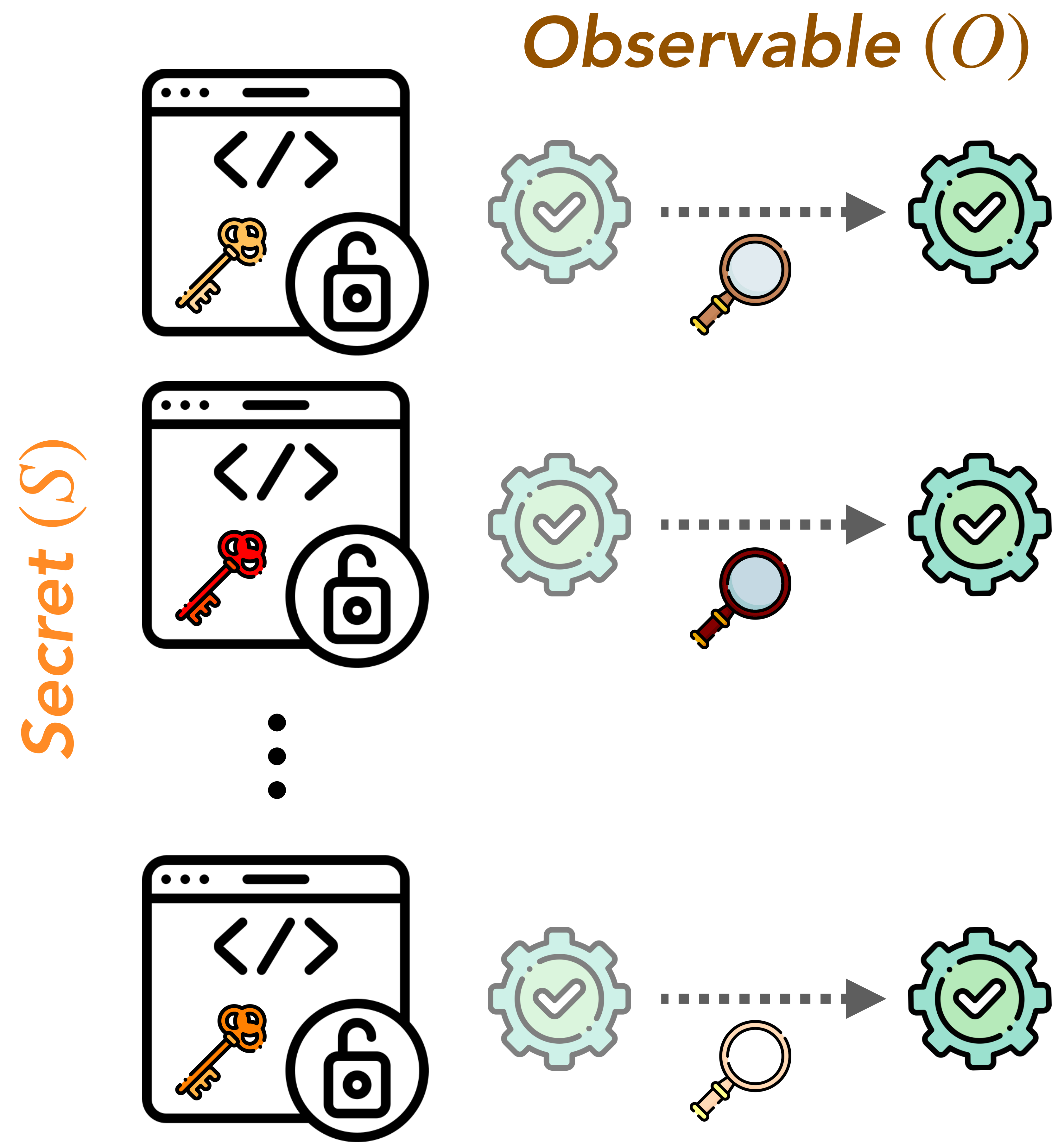


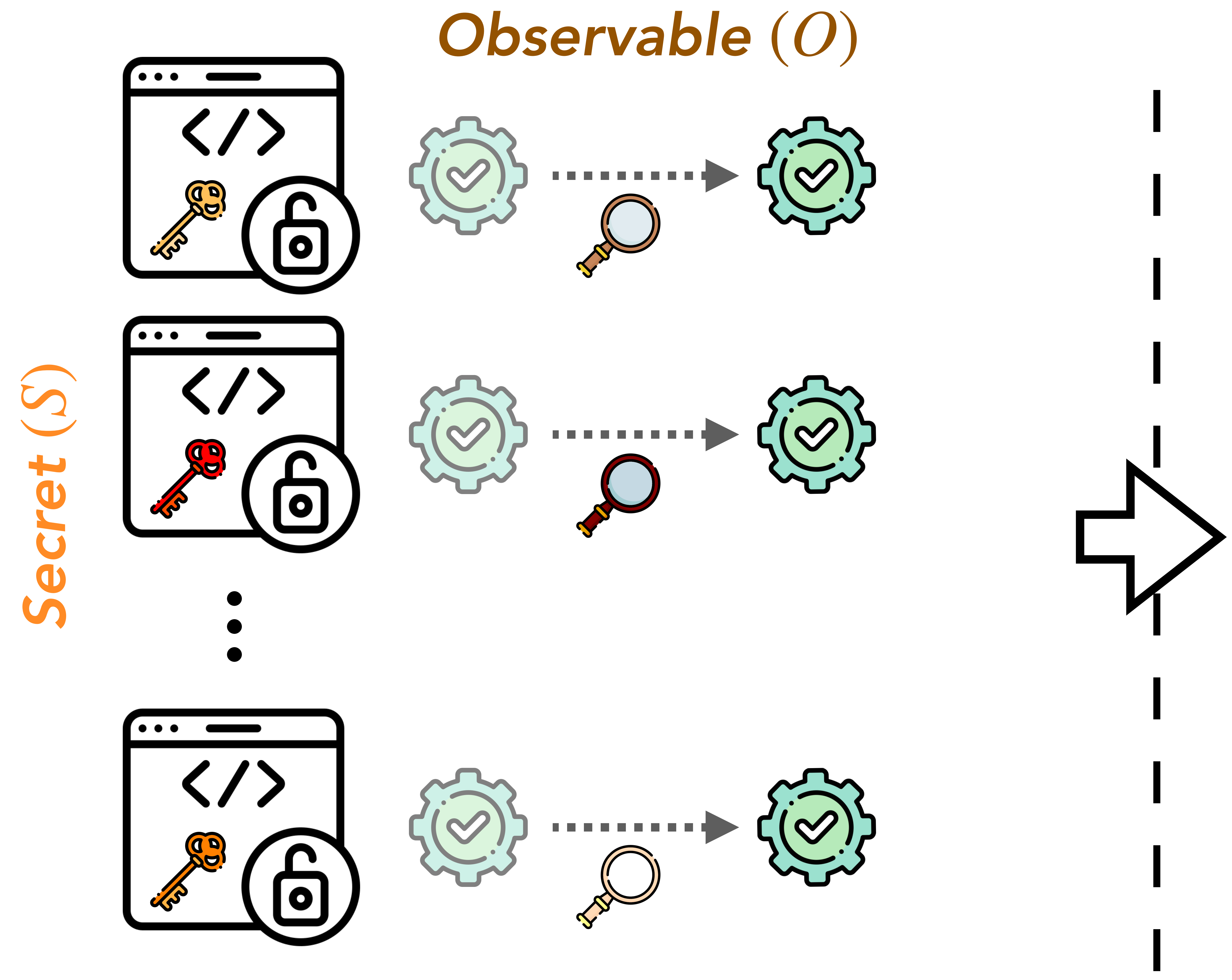
Observable (O)

Secret (S)

			...	
	100	0	...	30
	2	78	...	0
...	...	...	...	...
	13	50	...	50

***Sample* Joint Frequency Table**

(Existing) Empirical Information Leakage Analysis



Observable (O)

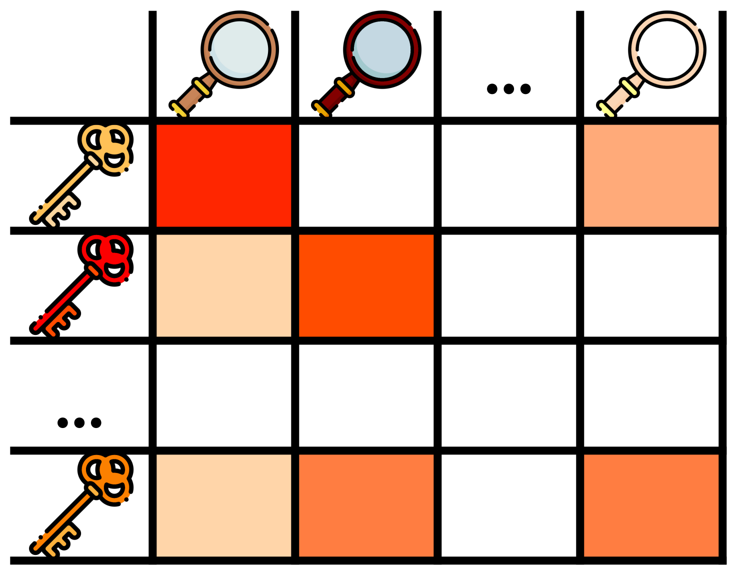
Secret (S)

	0.1	0.0	...	0.03
	0.002	0.078	...	0.0
...	...	...	...	...
	0.013	0.05	...	0.05

Empirical
Joint
Probability
Distribution

(Existing) Empirical Information Leakage Analysis

1. Empirical MI Estimator (**Empirical**)



Directly compute

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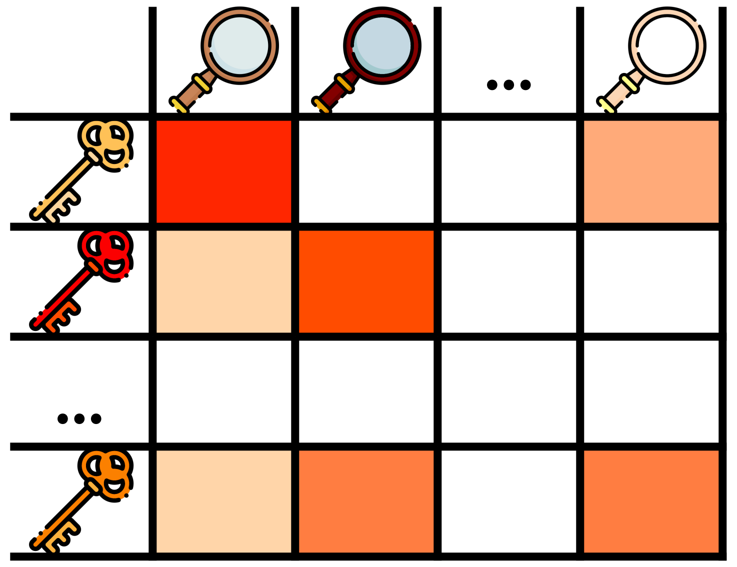

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Accuracy

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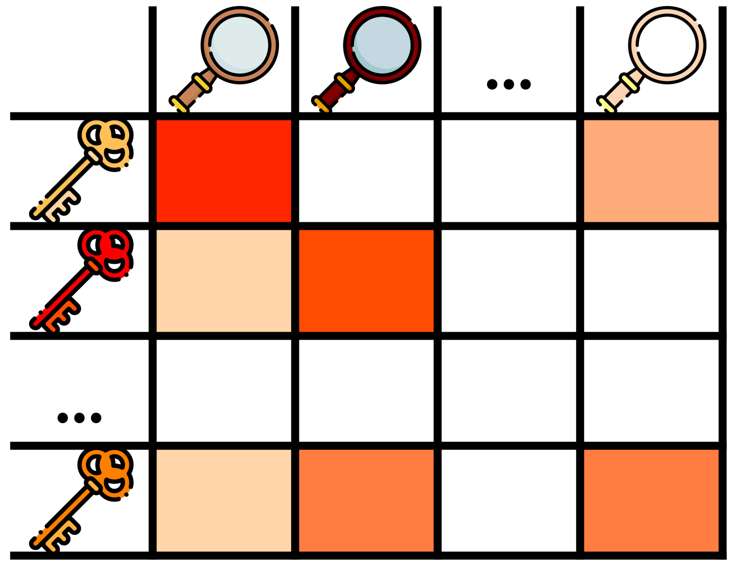

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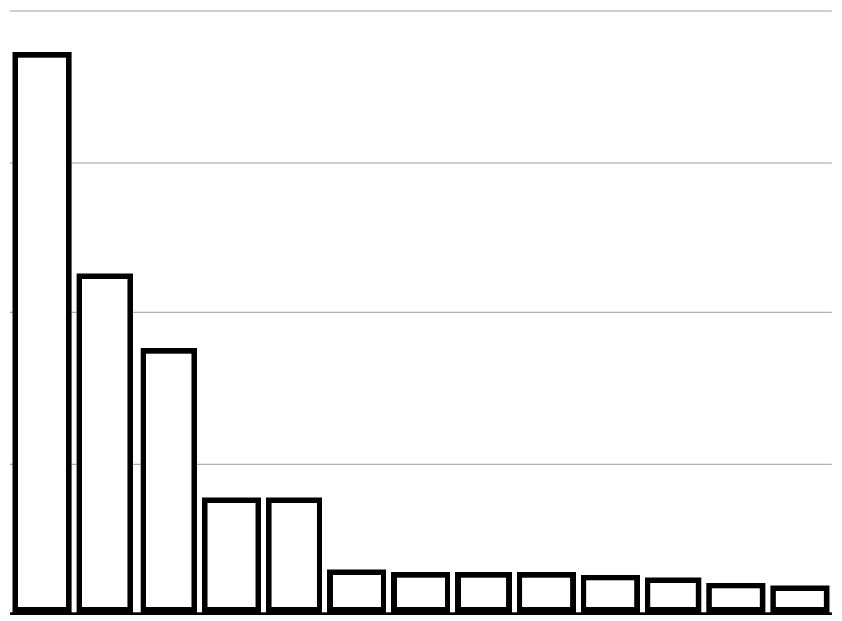
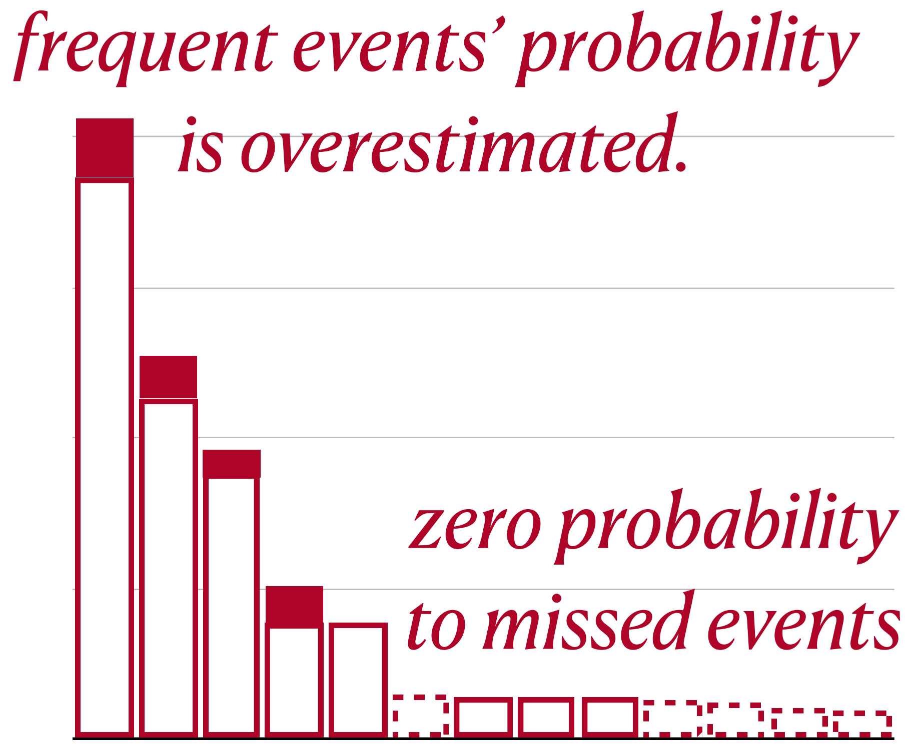


True Distribution



Empirical Dist.

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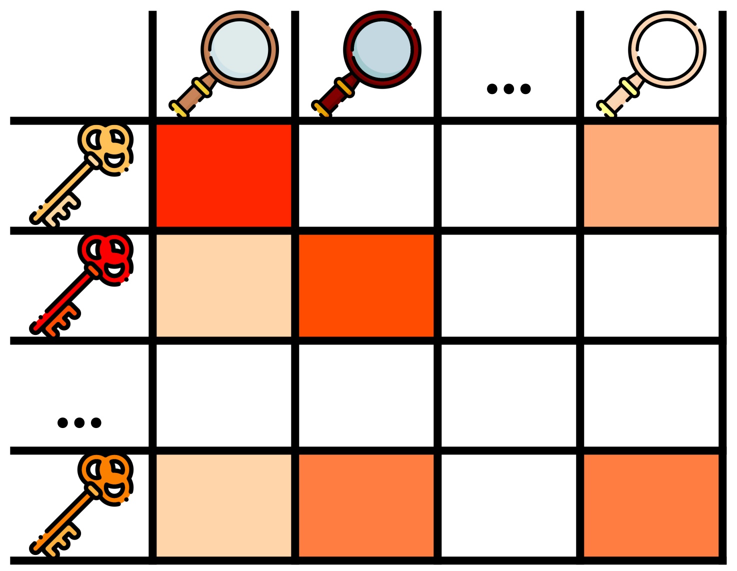

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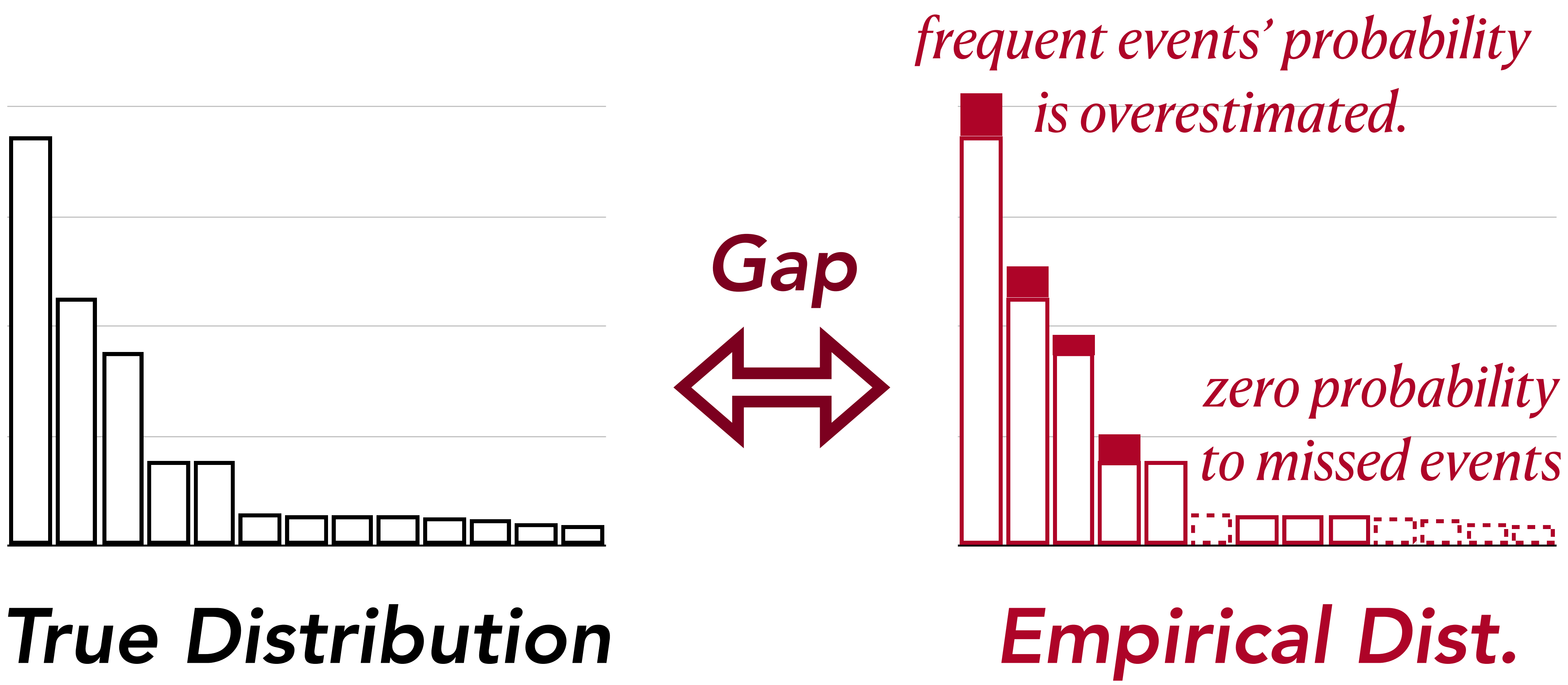

$$I \ll \hat{I}_{emp}$$

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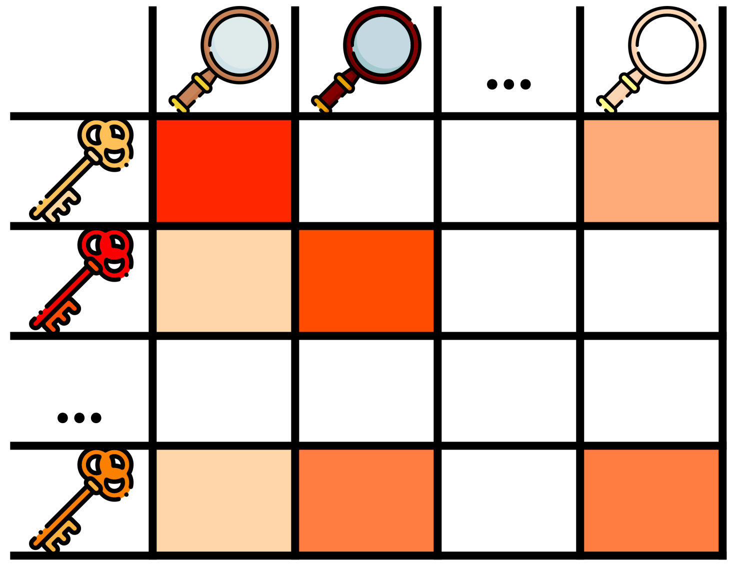

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The state-of-the-art estimator

# of unique sec. in the sample

# of unique obs. in the sample

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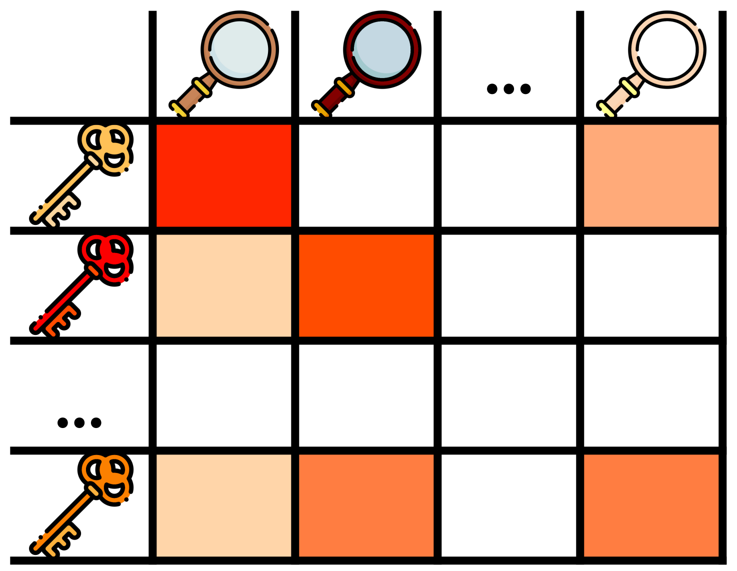

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memory access   time   program output

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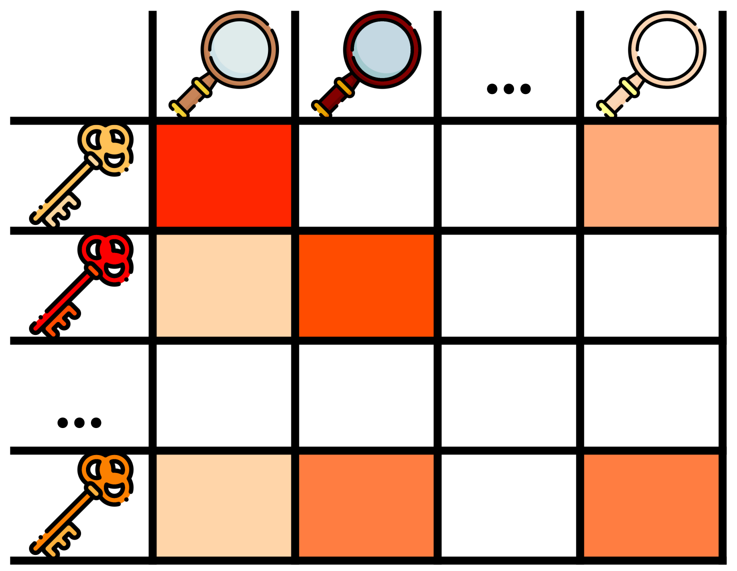

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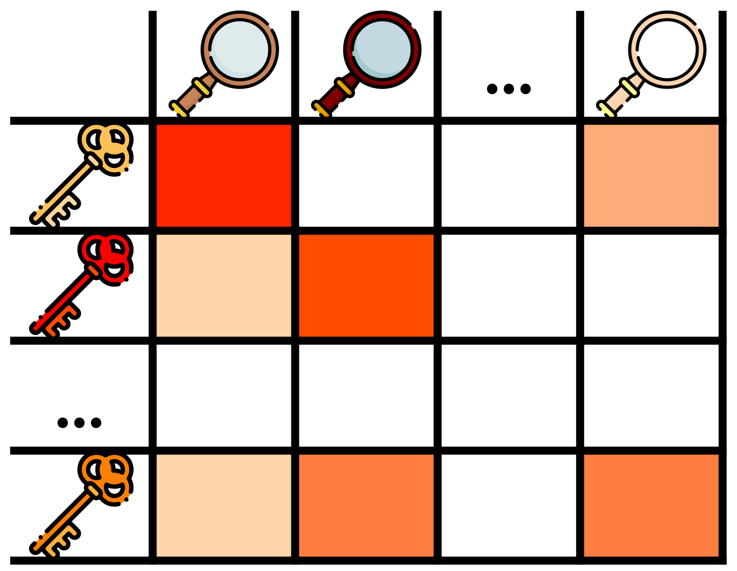

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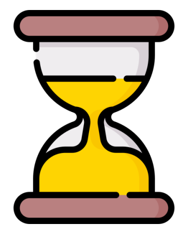

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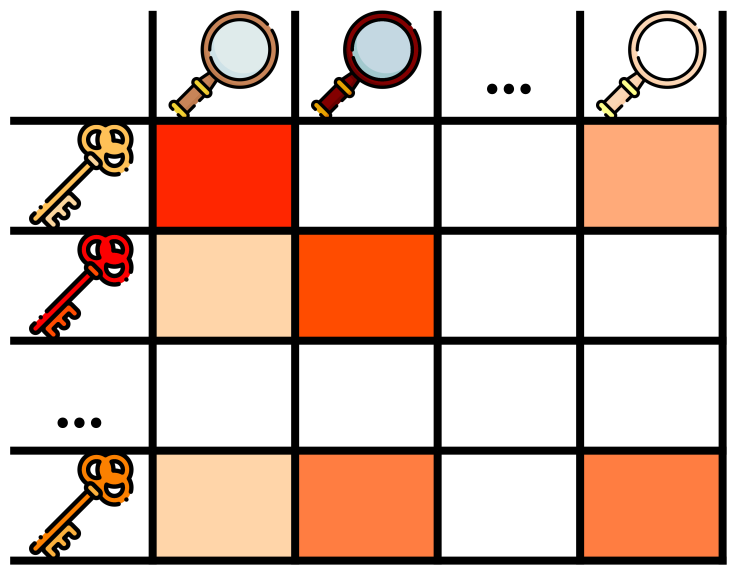

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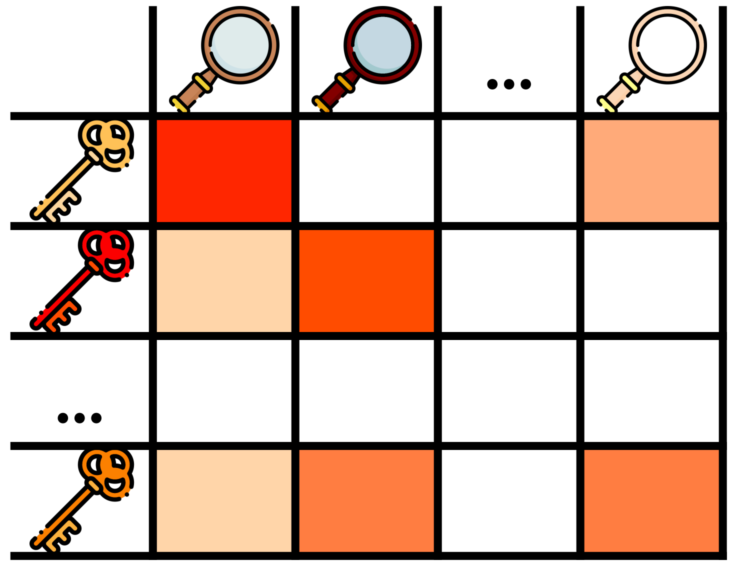

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underestimate

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Problem

Safety

It ***underestimates MI*** if there are ***rare events*** *in the sample*.

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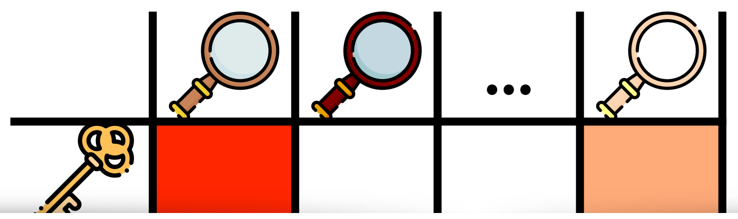
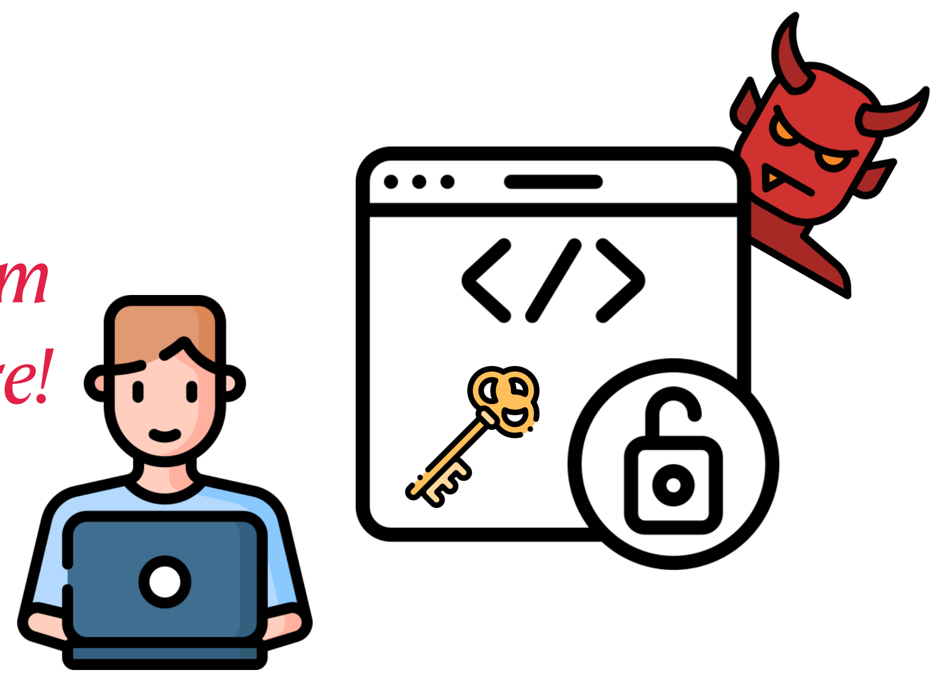


Underestimating the information leakage is especially ***harmful,*** since it leads to ***overconfidence in the privacy*** of the vulnerable software.



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underestimate   ***Bias correction term***

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Research **Aim**

Empirical estimator

$$\hat{I}_{emp}(S;O) = \hat{H}_{emp}(X) - \hat{H}_{emp}(X \mid Y)$$

Inaccurate

Miller estimator

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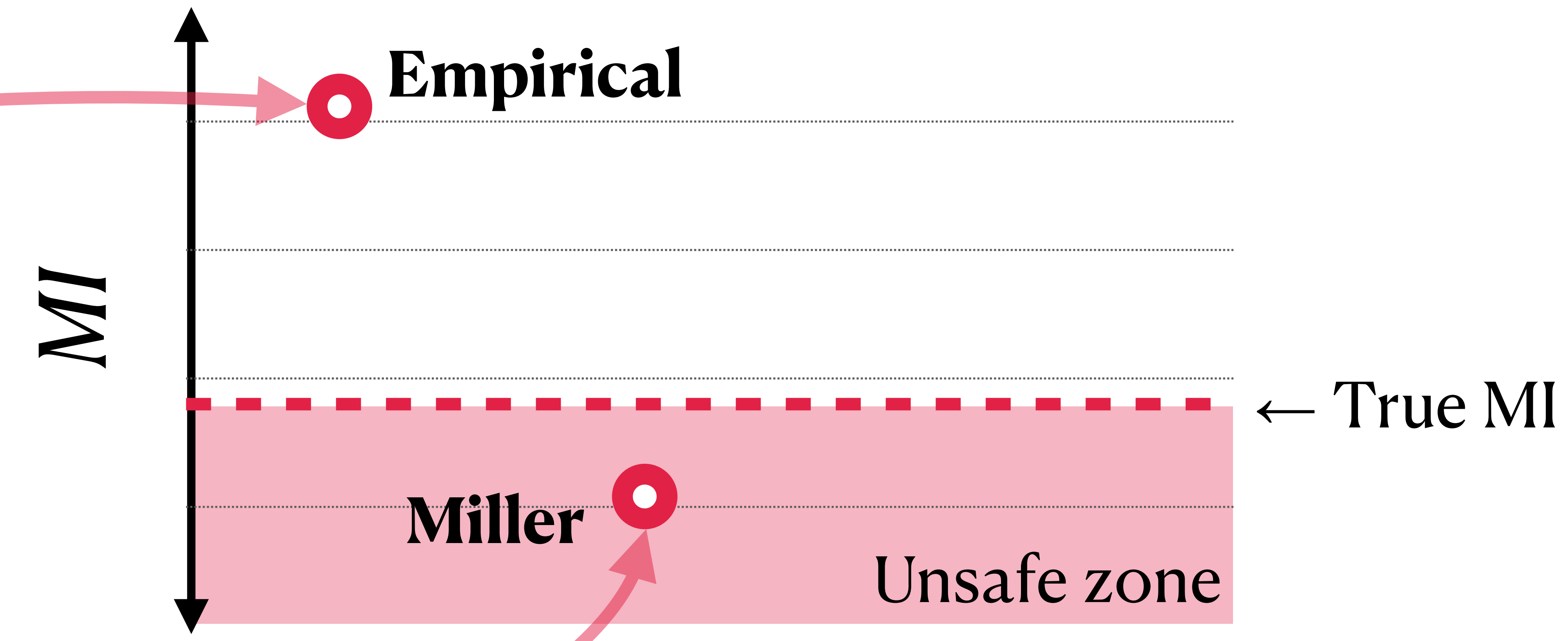
Unsafe w/ small samples



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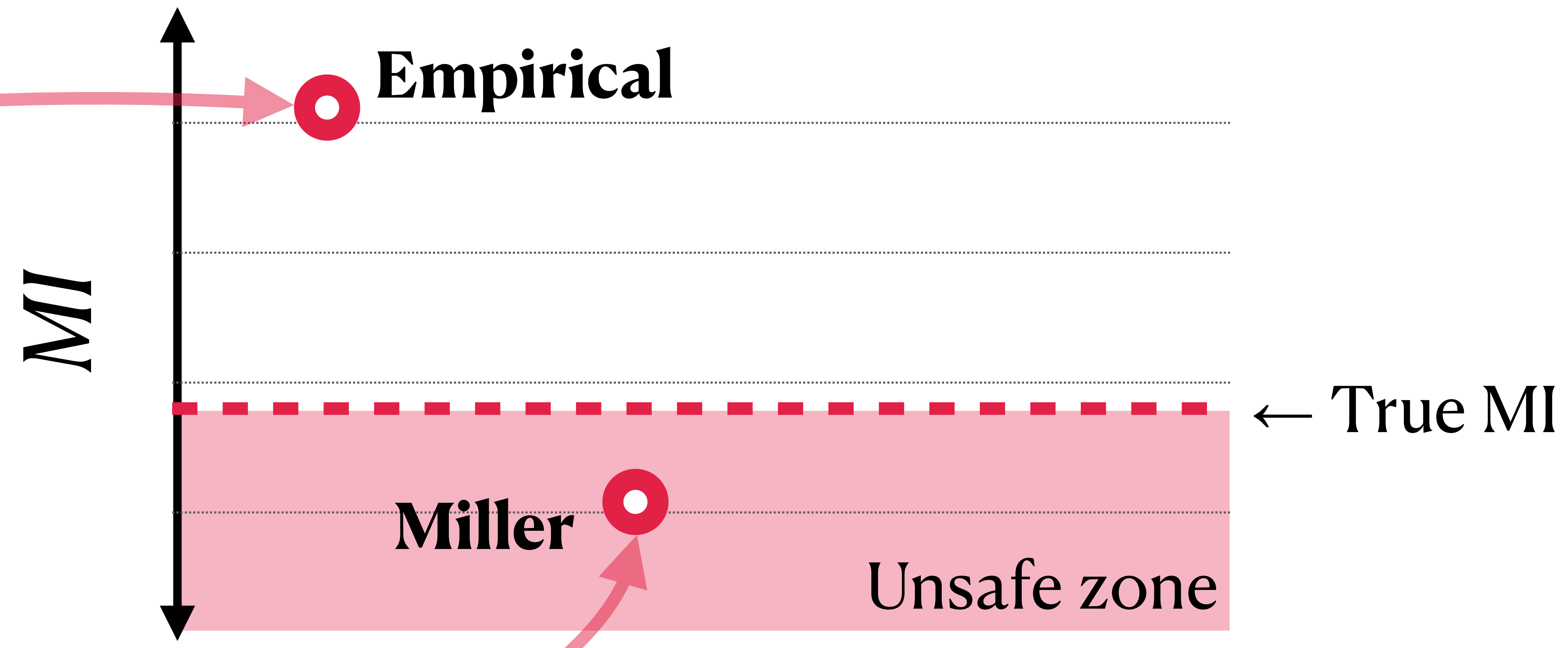

Existing estimators either produce ***inaccurate or unsafe*** *estimates due to mishandling* ***missing or rare events*****.**

Research Aim

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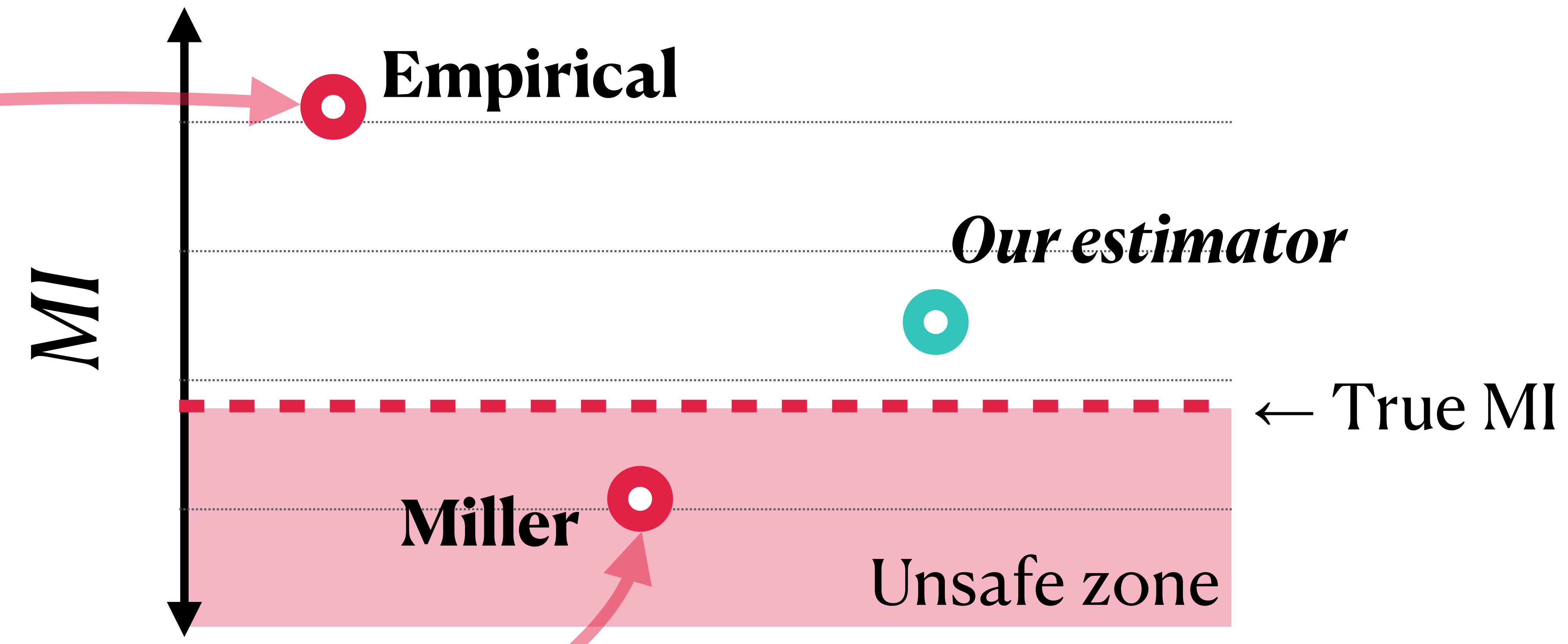

We developed an estimator that ***accurately*** *and* ***safely*** *estimates the leakage in the presence of* ***missing or rare events****.*

Chao's Multinomial Distribution (MD) Estimation

A. Chao et al, "Unveiling the species-rank abundance distribution by generalizing the good-turing sample coverage theory." Ecology, vol. 96 5, pp. 1189–201, 2015.

Chao's Multinomial Distribution (MD) Estimation

- Given samples from the unknown multinomial distribution (MD), it **reconstructs the underlying MD** by approximation.

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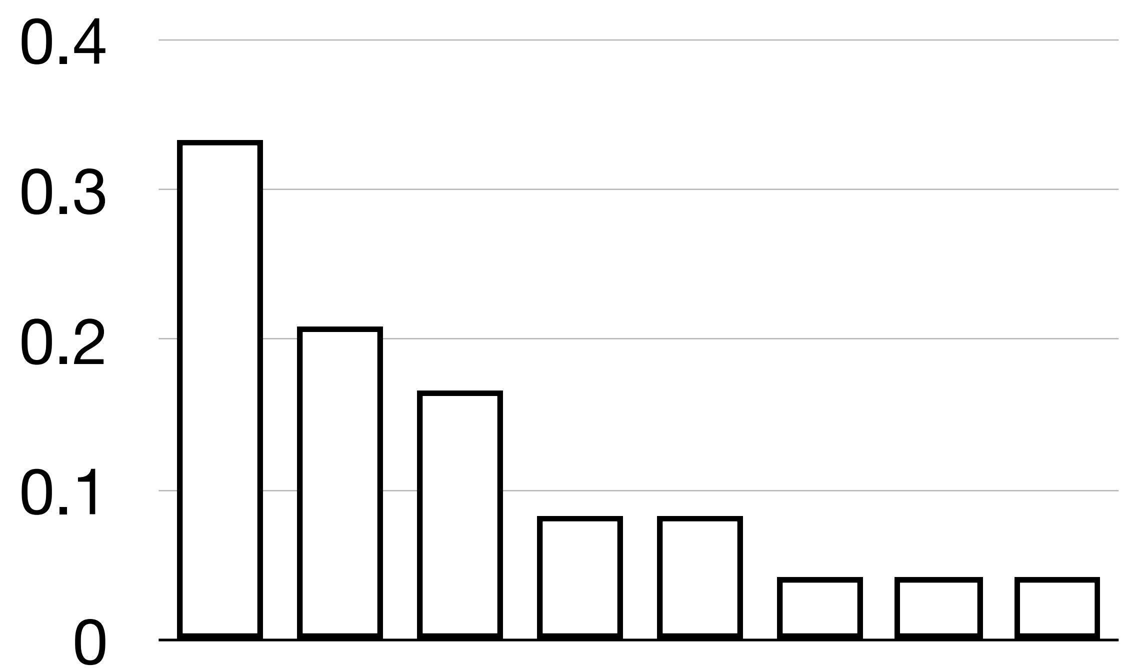

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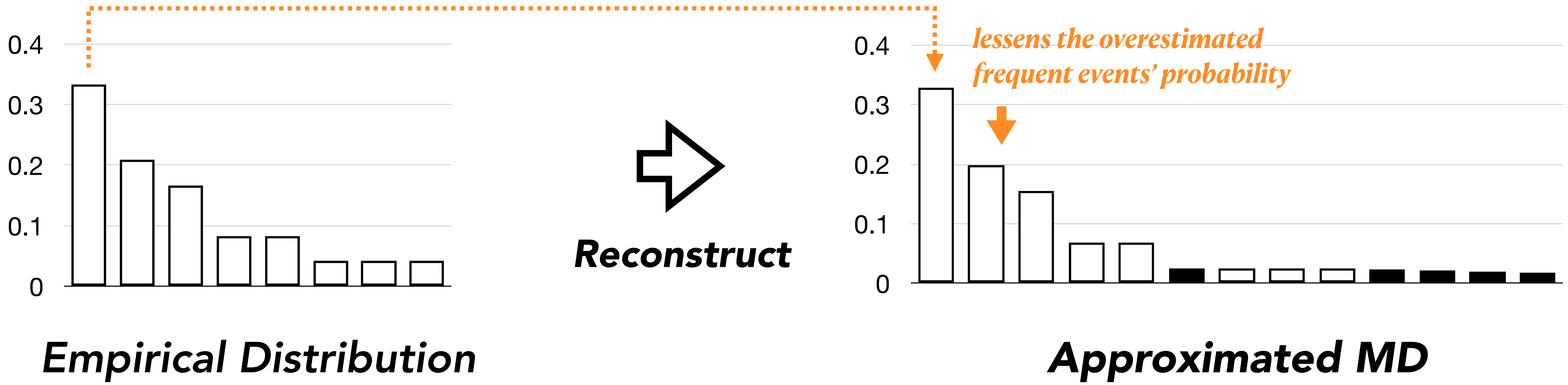

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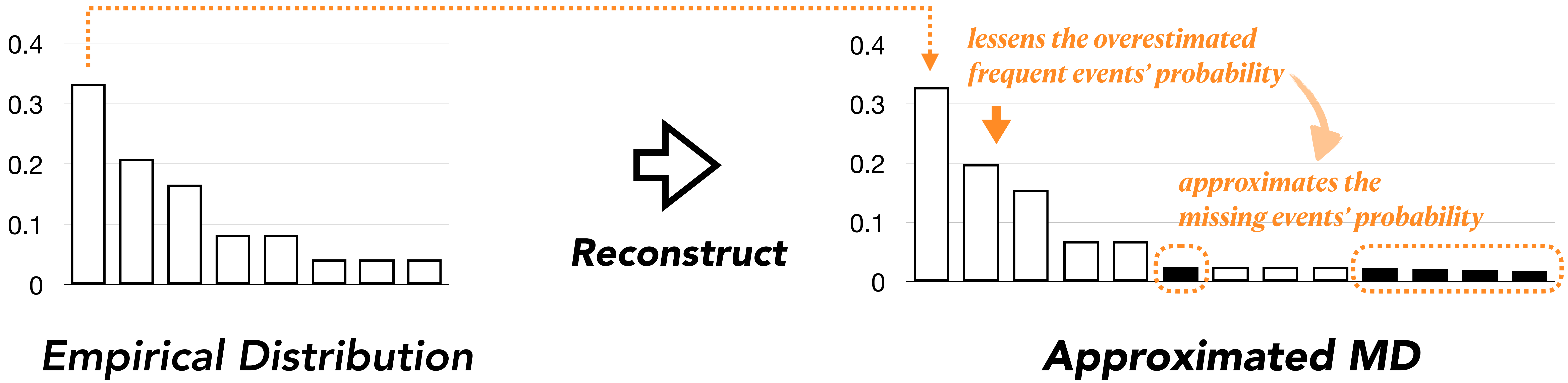

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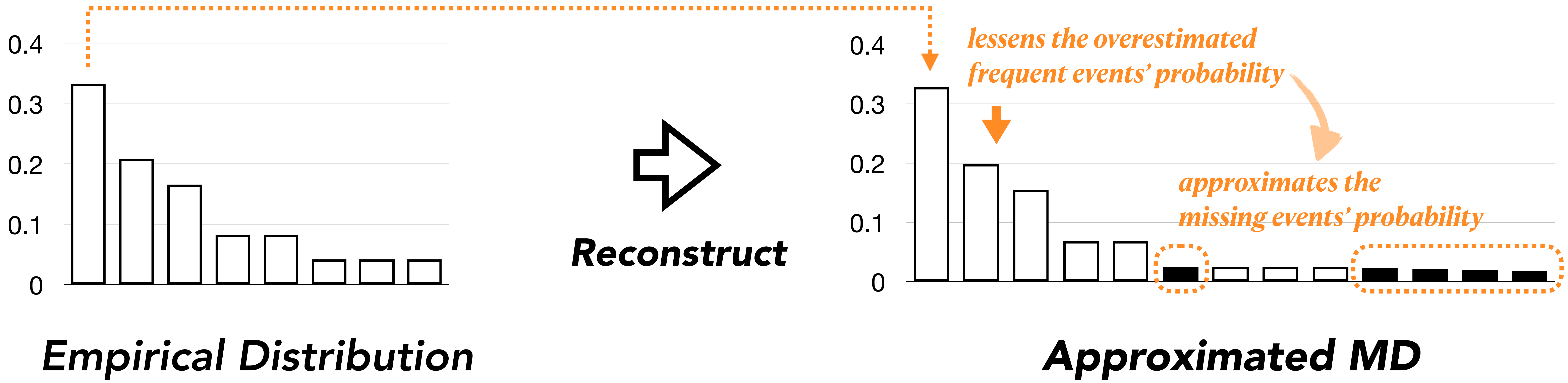
➡ *Handle the missing/rare events problem* 😀



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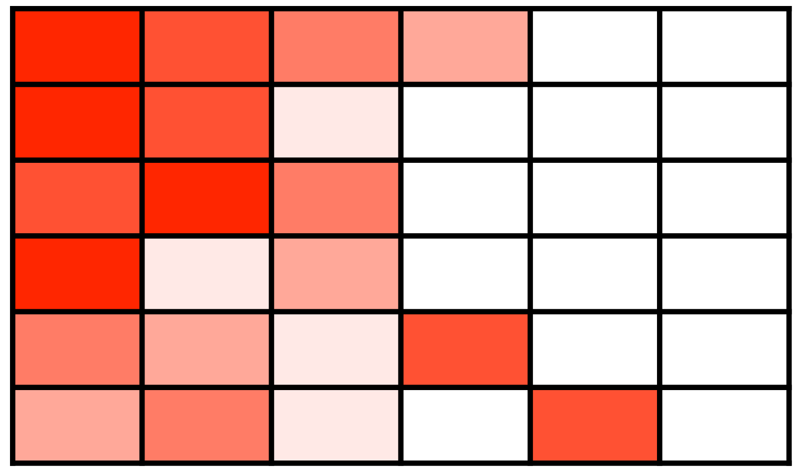
Challenge of apply MD estimation for MI estimation



Empirical Joint
Probability Dist.

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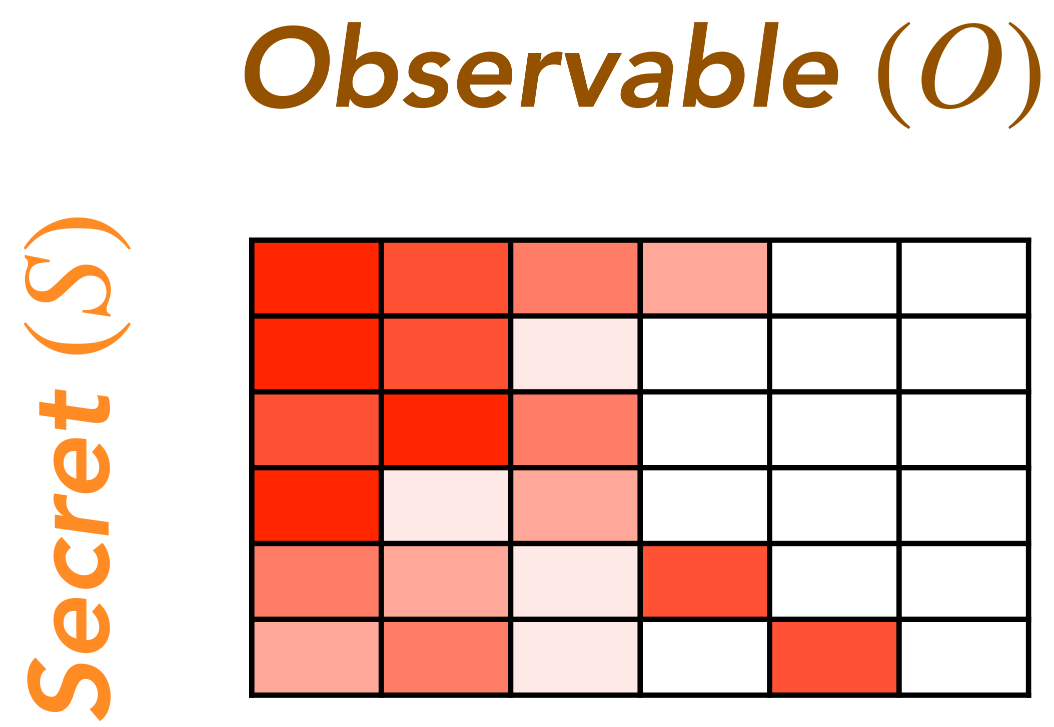

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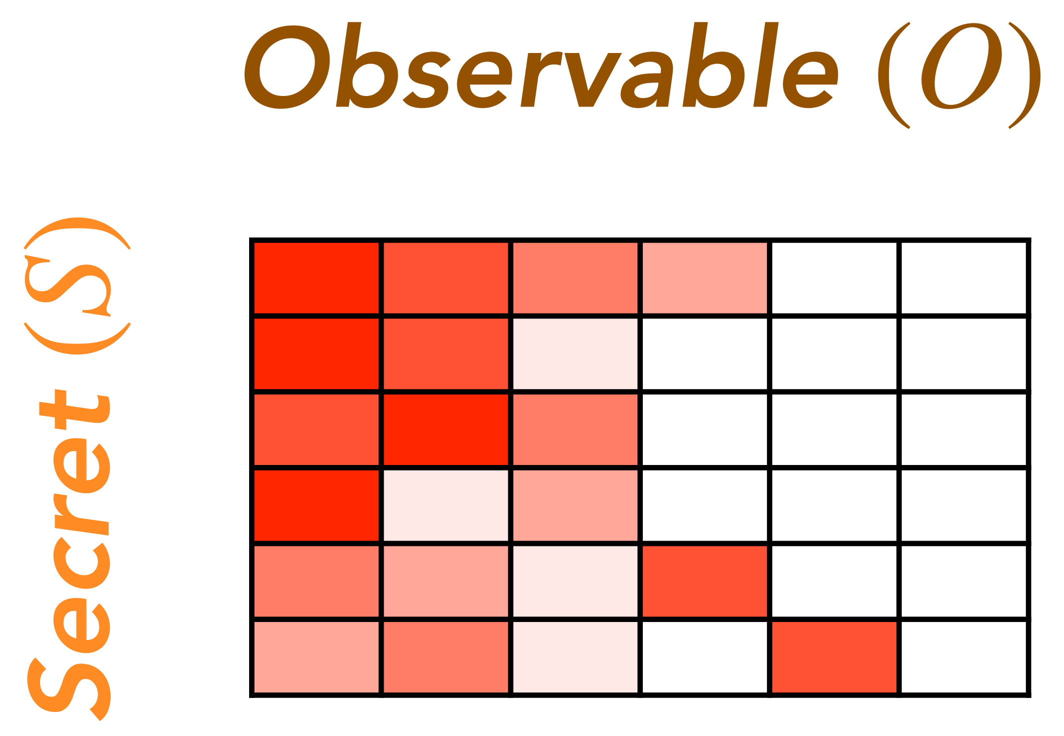

Empirical Joint Probability Dist.

Challenge 1.

MD estimation is for ***a single random variable***, while **MI estimation** needs to handle ***two random variables.***

Our Approach to estimate MI

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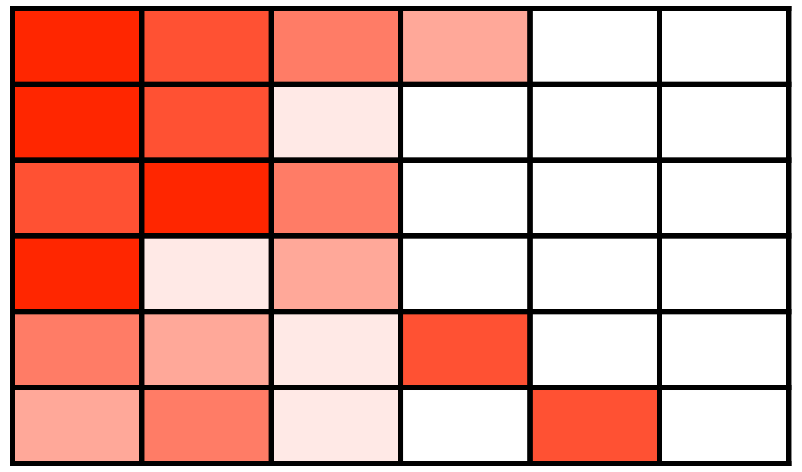
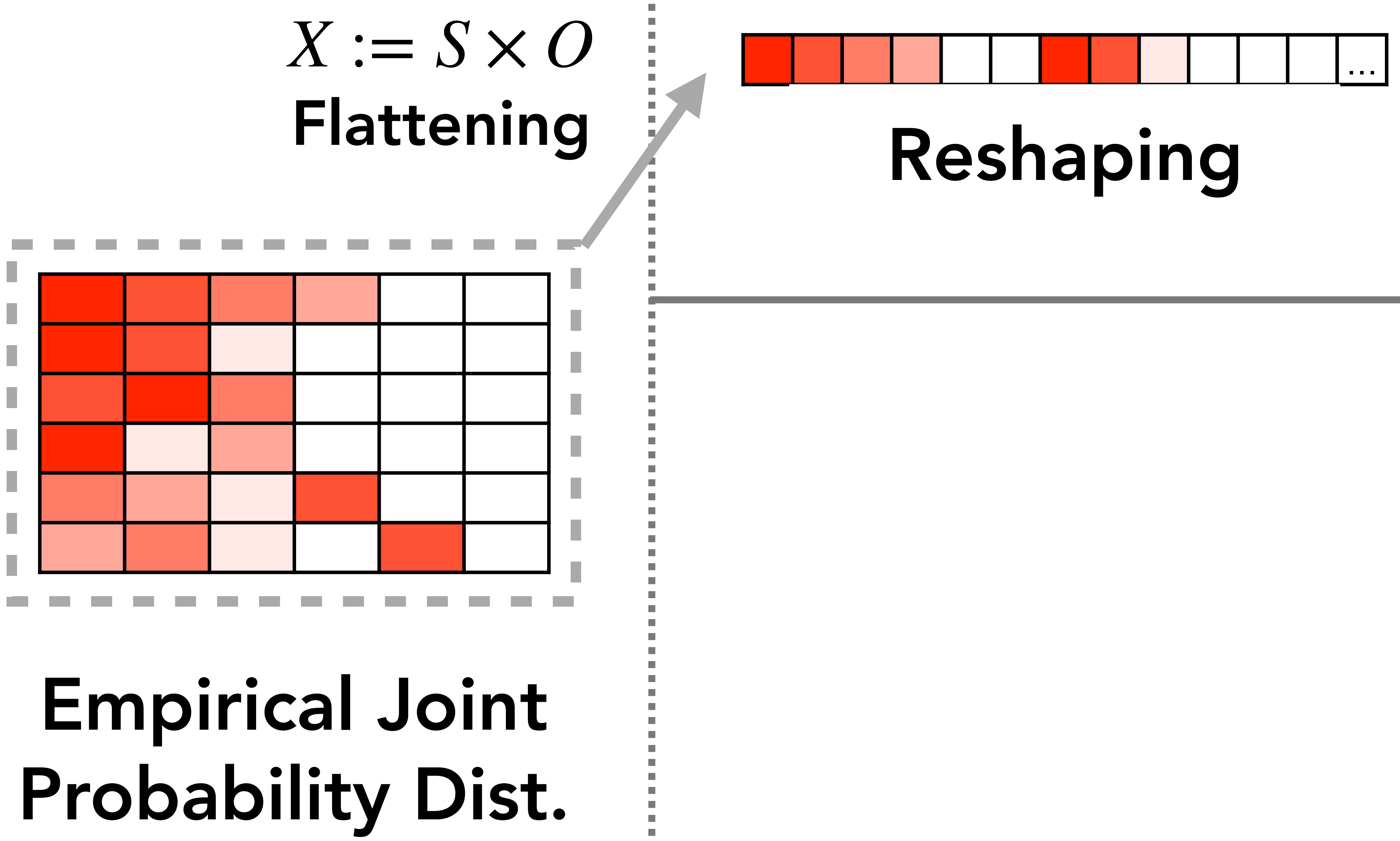


Empirical Joint Probability Dist.

Our Approach to estimate MI

1. Flatten



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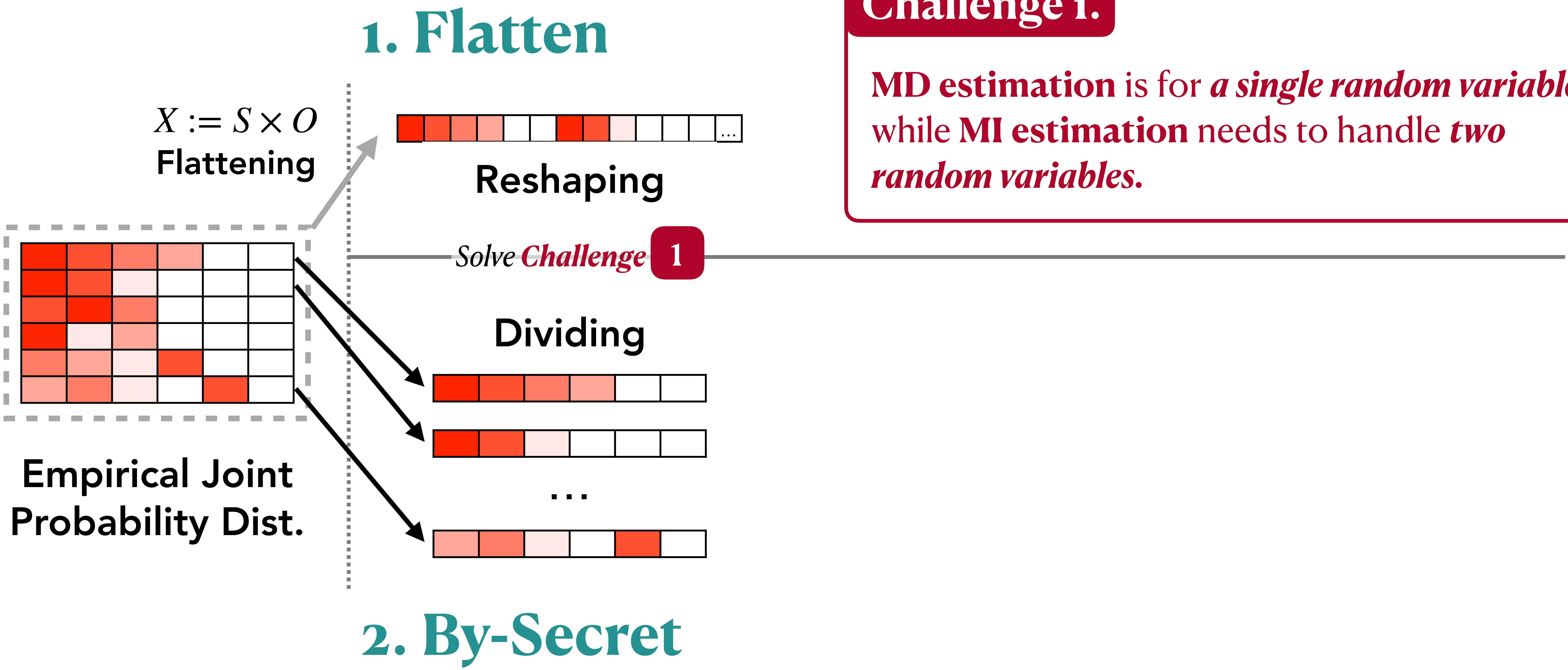

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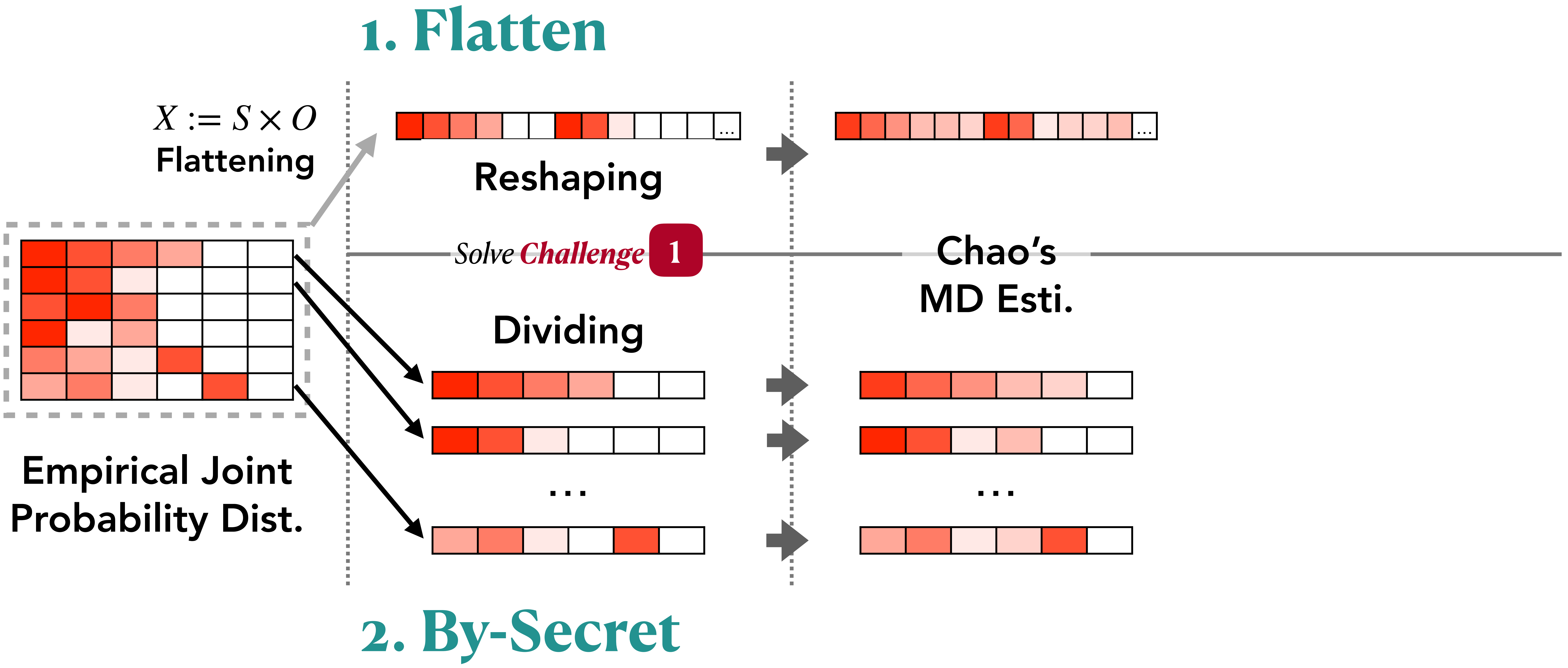

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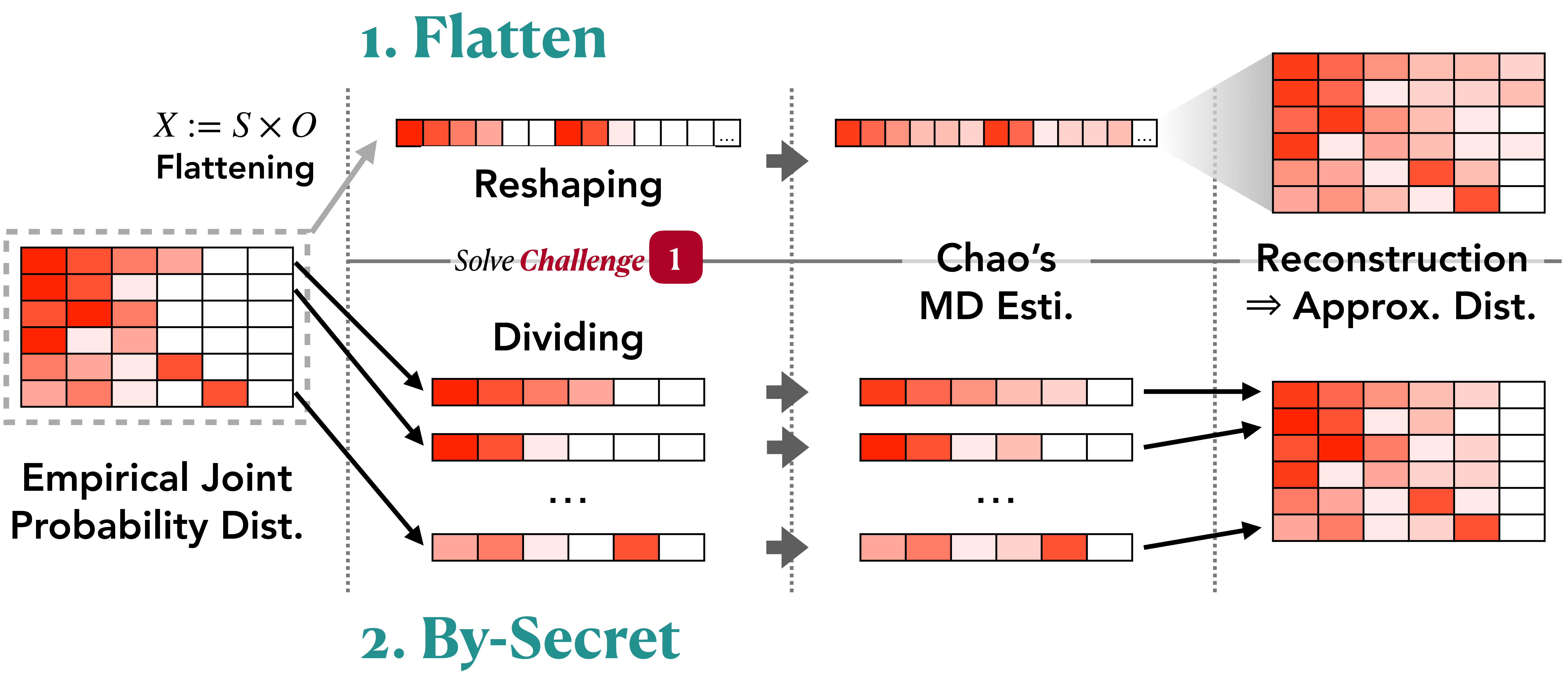
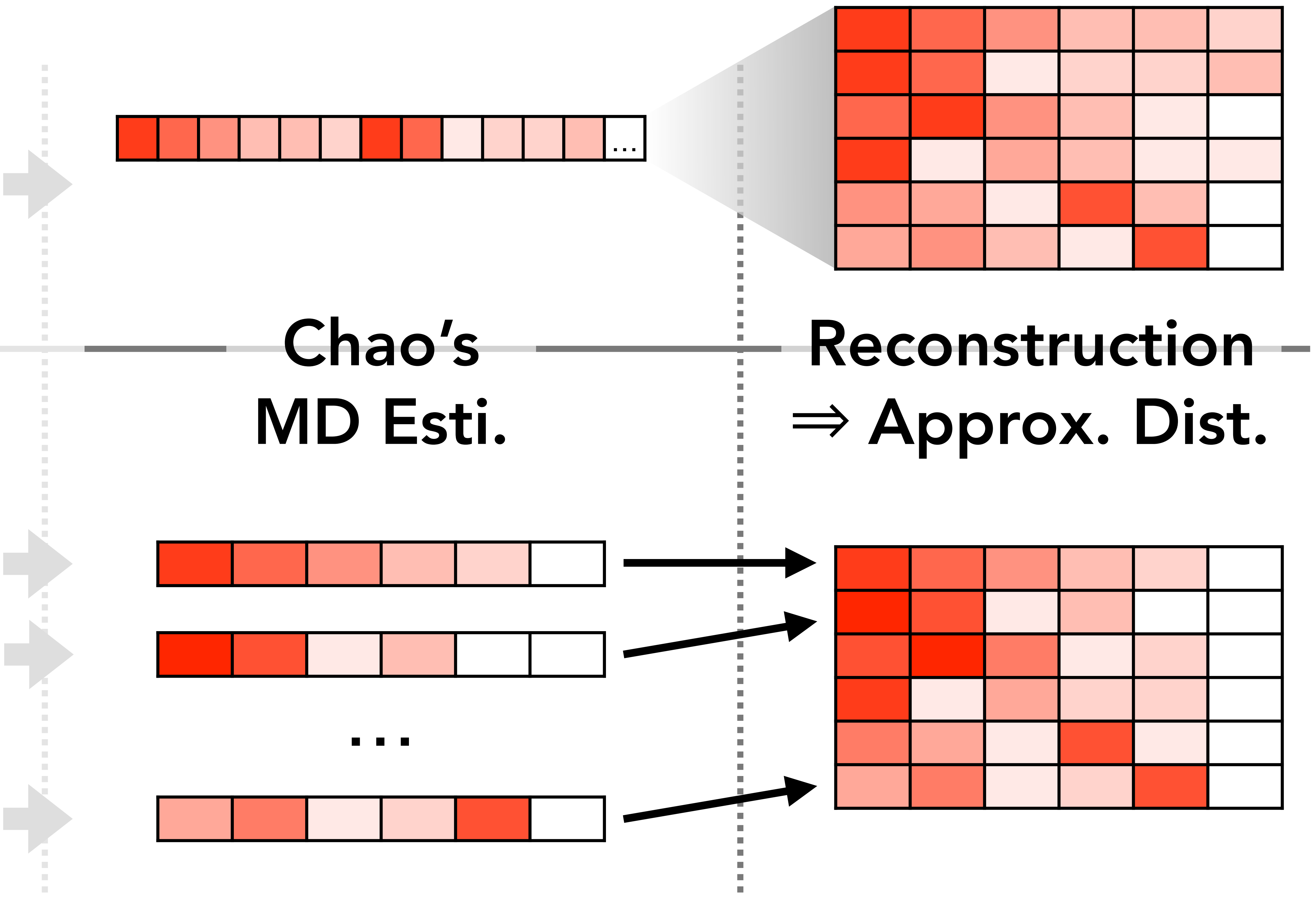


Our Approach to estimate MI

Challenge 2.

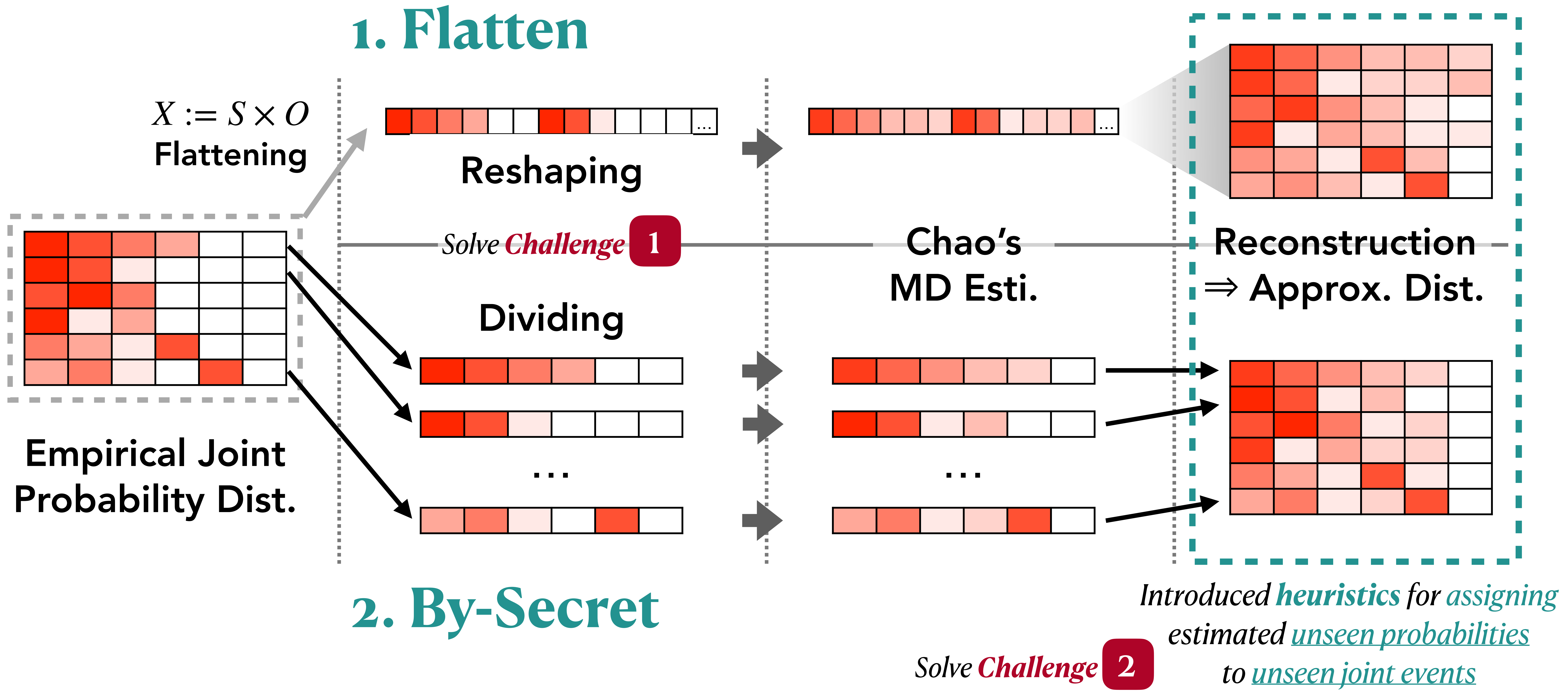
MD estimation does not provide **which missing event** has **which probability**.

It only estimates the ***shape of the distribution****, not the probability for each event.*



2. By-Secret

Our Approach to estimate MI



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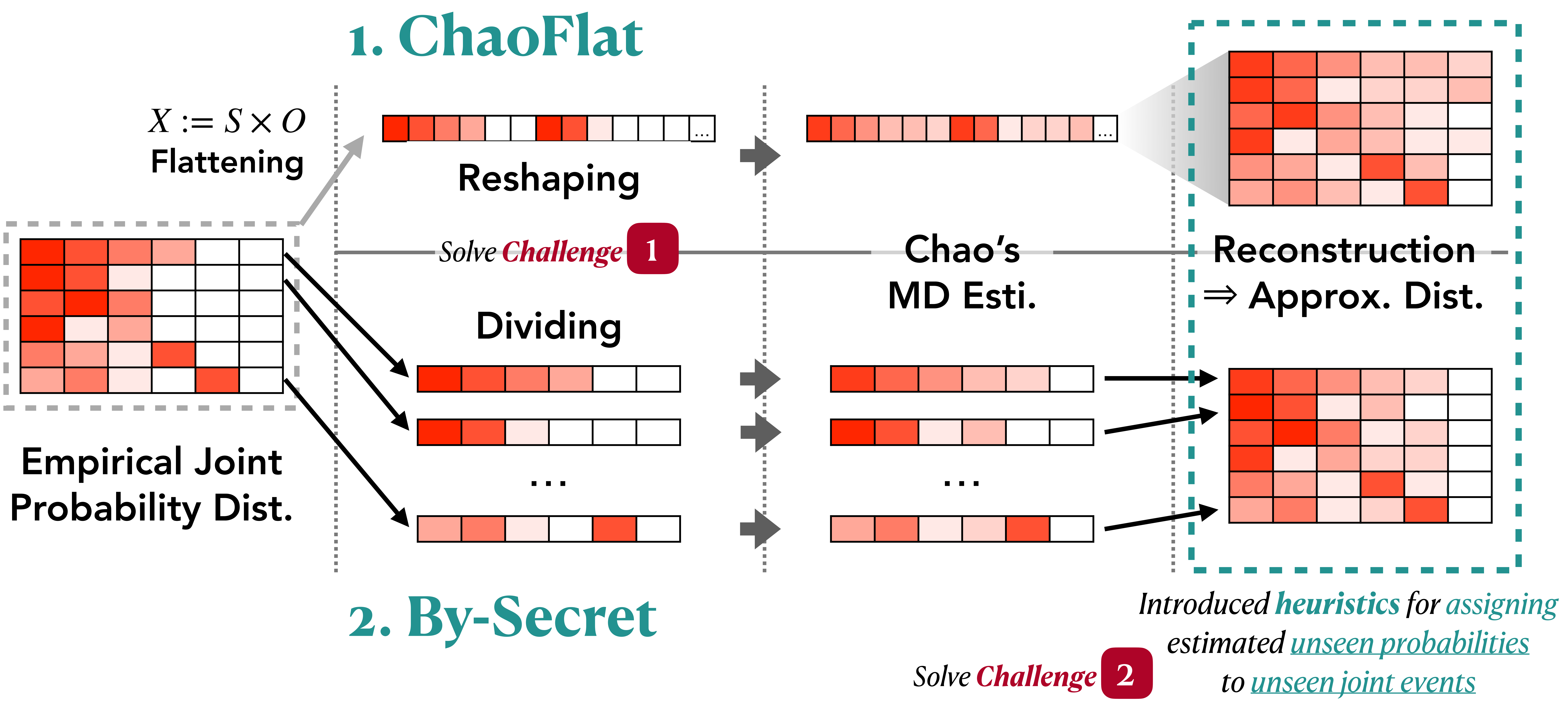

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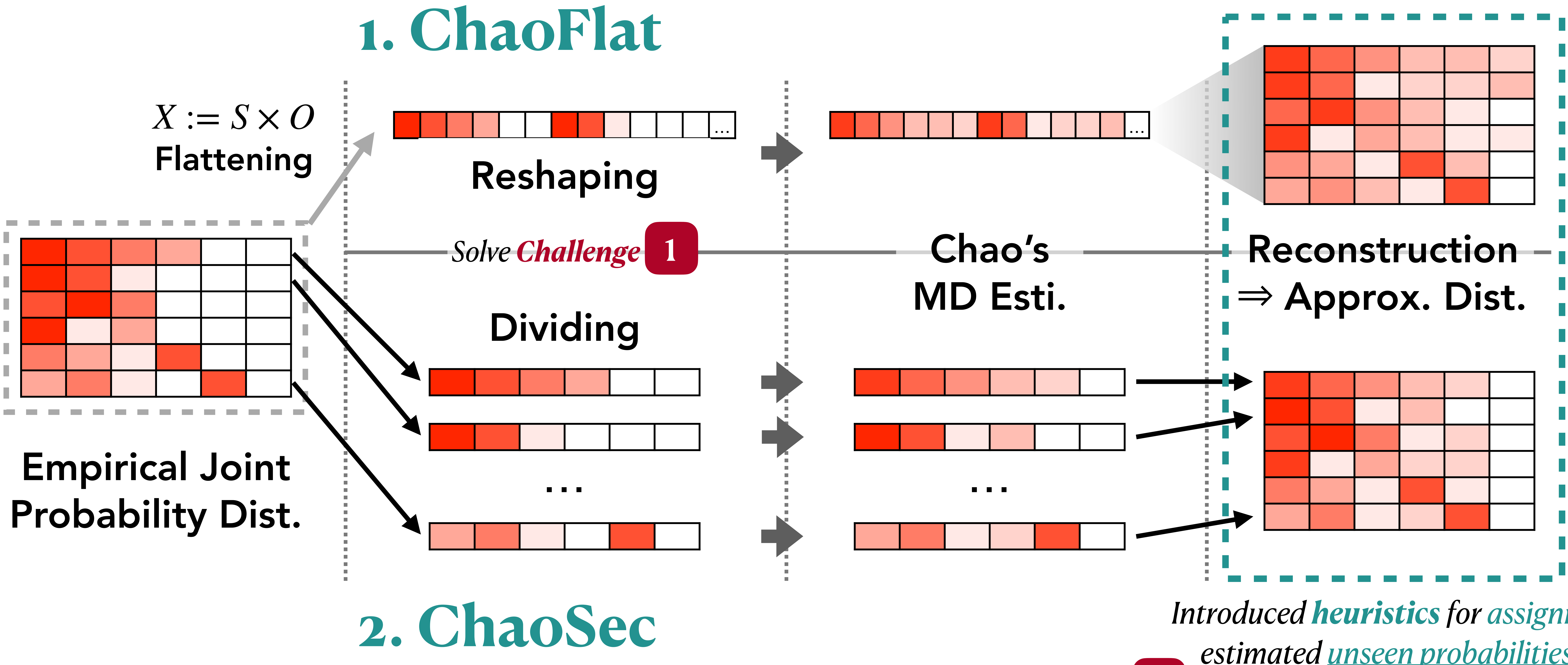

Introduced ***heuristics*** *for assigning estimated unseen probabilities to unseen joint events*

Solve ***Challenge*** **2**

Evaluation

Our estimator

ChaoFlat **ChaoSec**

VS

Baselines

Empirical **Miller**

Evaluation

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VS

- Accuracy (*Mean Square Error)*
- Safety (*whether underestimate)*

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Benchmark

1. Subject programs from previous study

Subject	$(\lvert\mathcal{X}\rvert, \lvert\mathcal{Y}\rvert)$	Variants (N)
ProbTerm	$(N+1, 10\text{–}20)$	$\{5, 7, 9, 12\}$
RandomWalk	$(500, 24\text{–}40)$	$\{3, 5, 7, 14\}$
Reservoir	$(2^N, 2^{N/2})$	$\{4, 6, 8, 10, 12\}$
SmartGrid	$(3^N, 12)$	$\{1, 2, 3, 4, 5\}$
Window	(N, N)	$\{20, 24, 28, 32\}$

- **Small size**
- Known ground-truth MI

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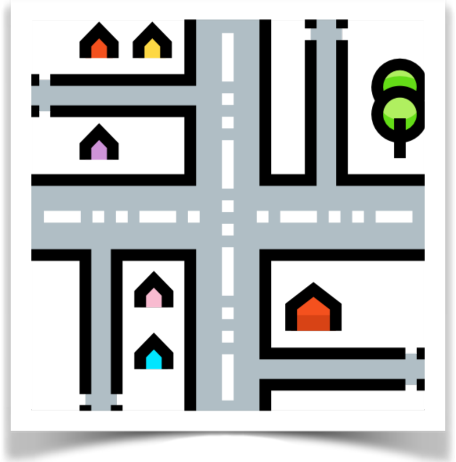
2. Practical scenarios with real-world examples

1) 

Location Privacy

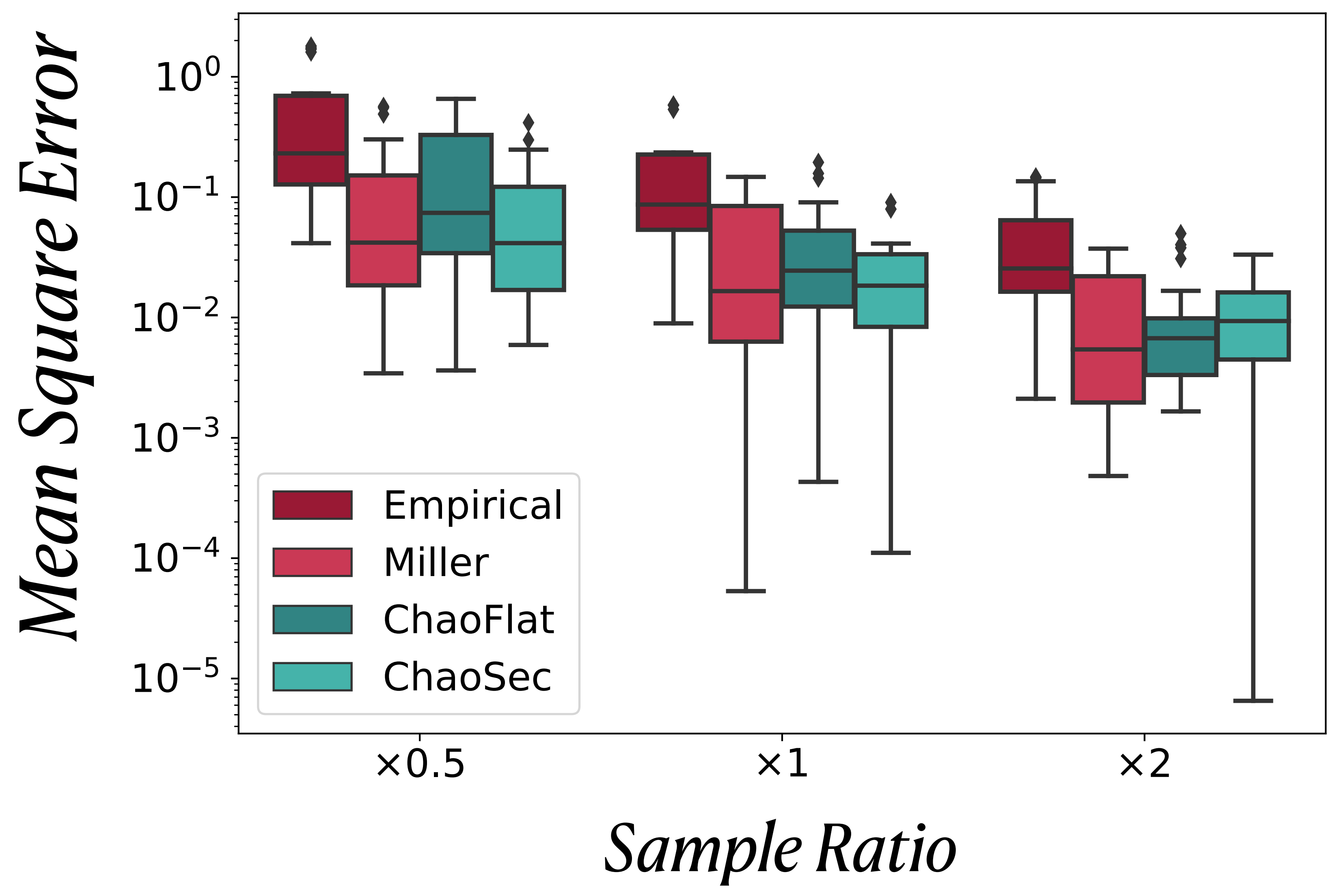
2) 

Passport Tracing

- **Domain of the joint event space ⟨ , ⟩ is substantially larger**
- Empirical ground truth

Result 1: Subject Programs from Prev. Study

— where the observable space is small —



* *Sample Ratio of* $\times k$: $|\text{sample}| = |S| \cdot |O| \times k$

Accuracy

- MSE(**Empirical**) $\gg$ MSE(**Miller**), MSE(**ChaoFlat**), MSE(**ChaoSec**)
- No significant difference b/w **Miller**, **ChaoFlat**, **ChaoSec**

Result 1: Subject Programs from Prev. Study

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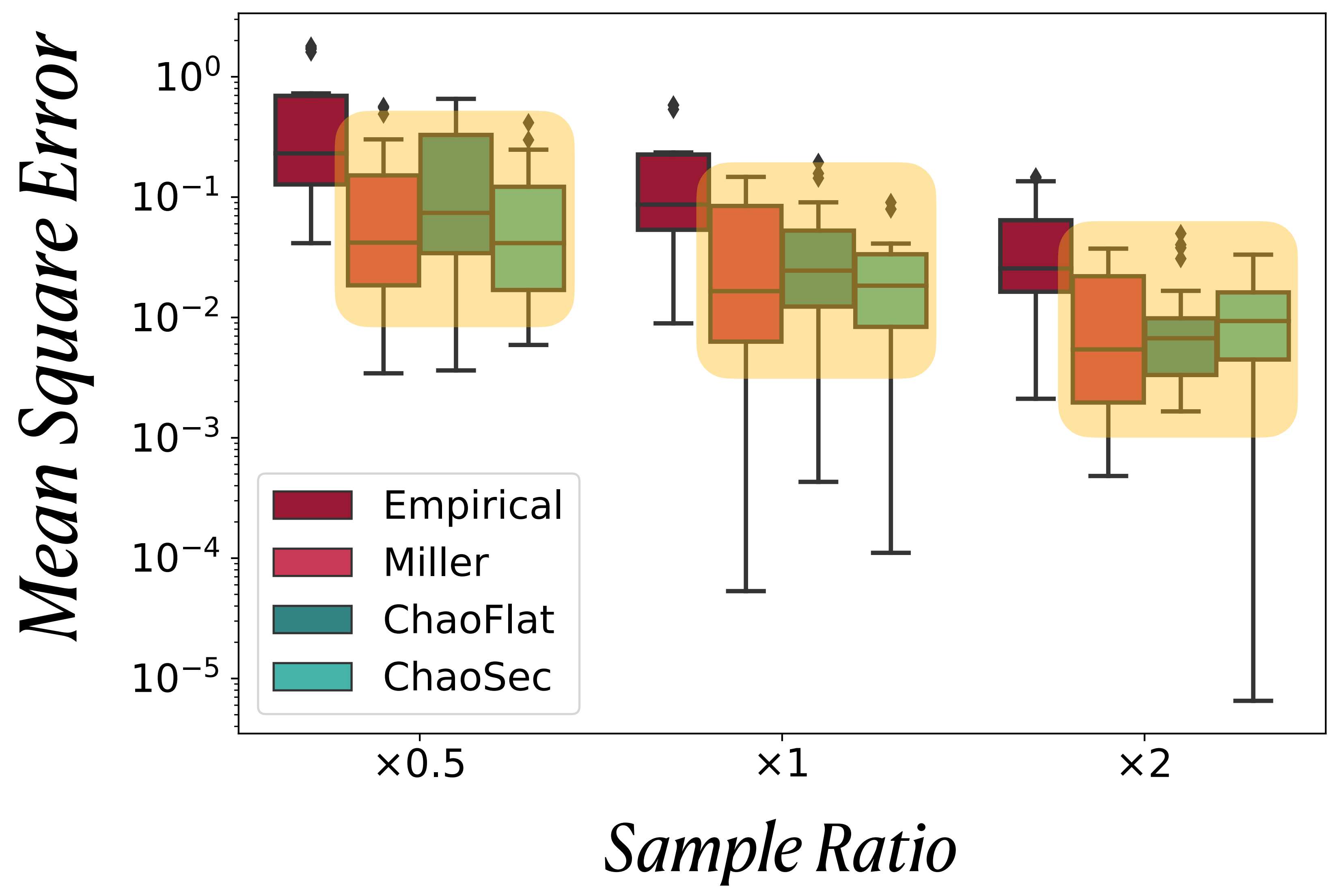

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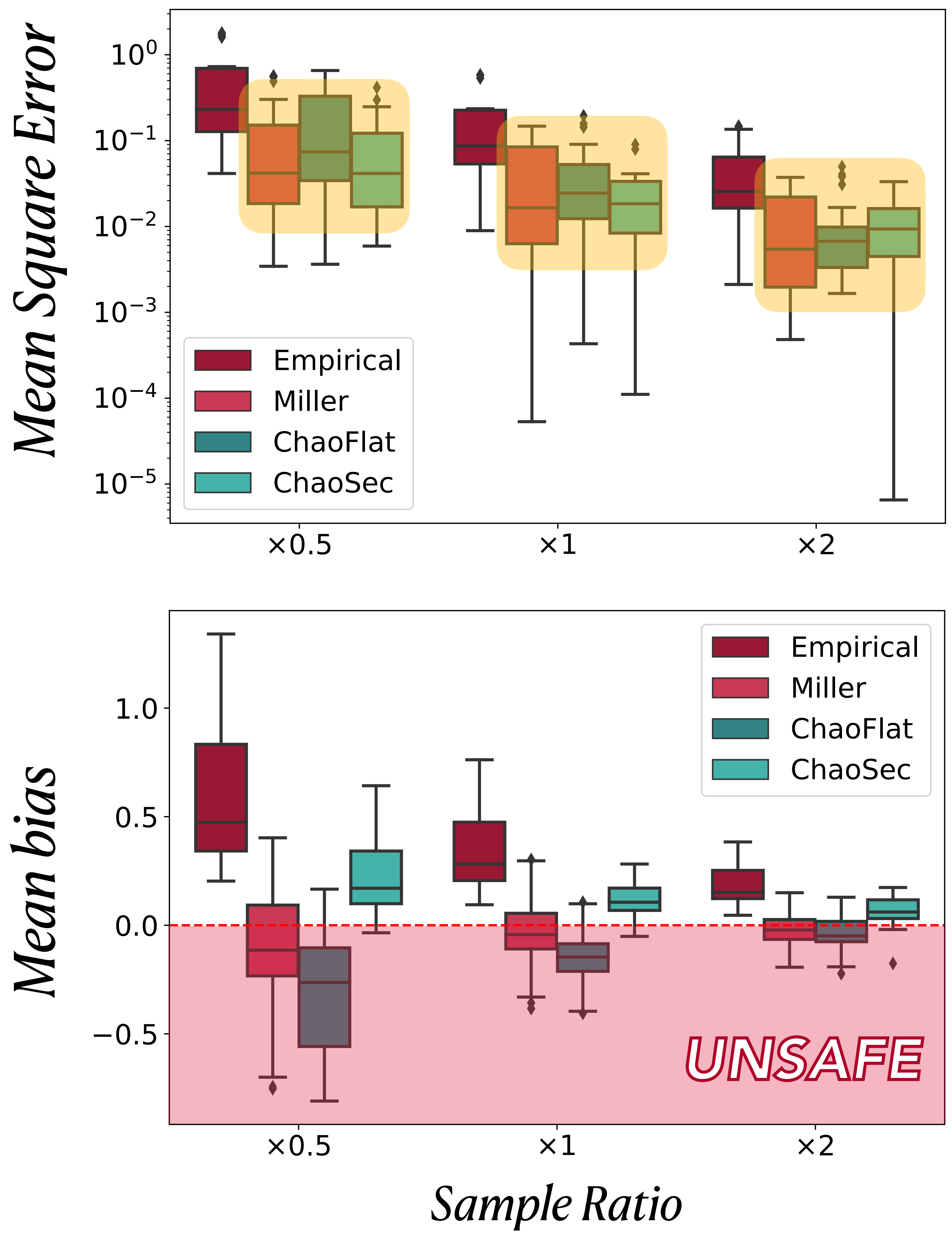


Accuracy

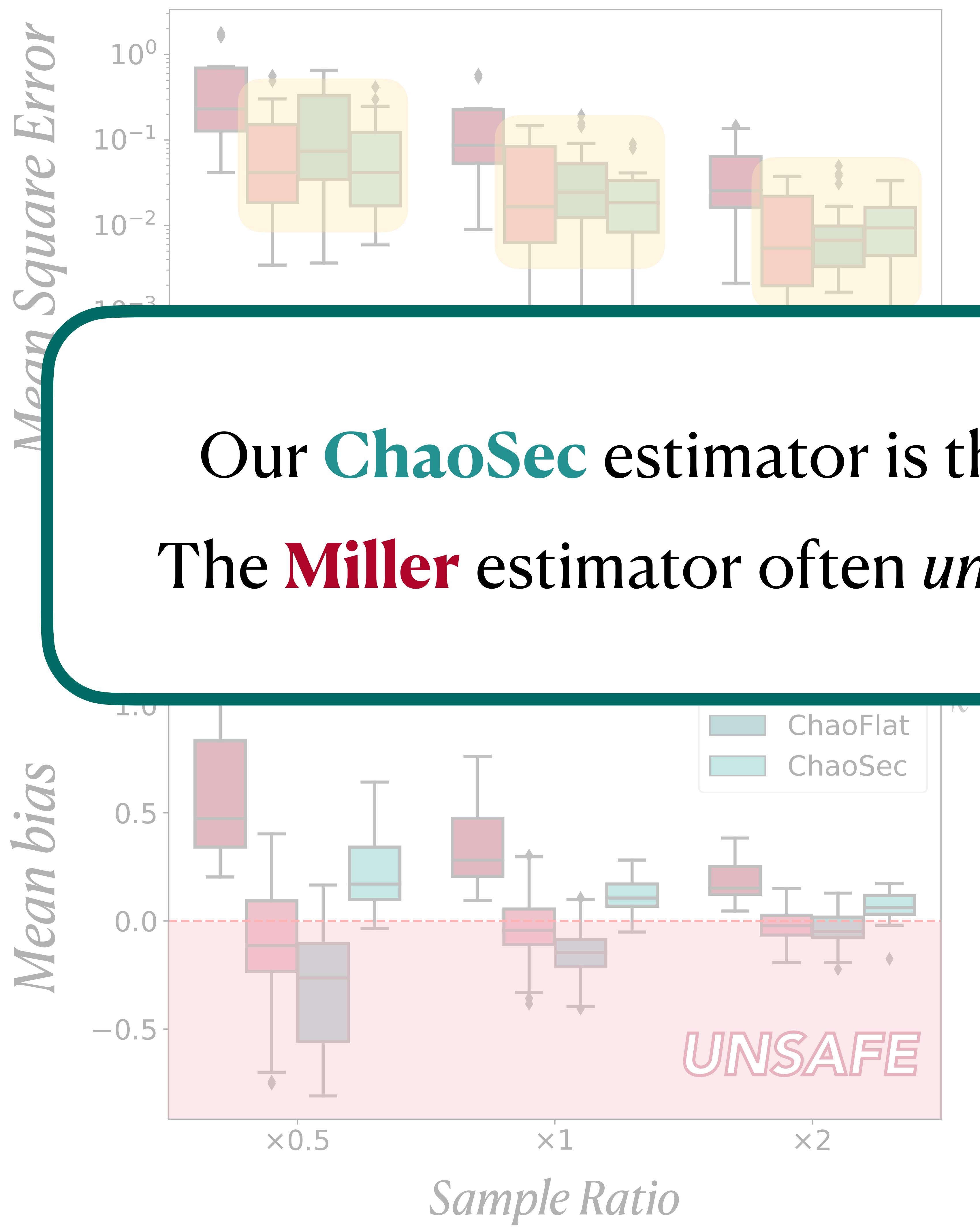
- MSE(**Empirical**)
  $\gg$ MSE(**Miller**), MSE(**ChaoFlat**), MSE(**ChaoSec**)
- No significant difference b/w **Miller, ChaoFlat, ChaoSec**

Safety

- **Miller** underestimates **57%** of the estimation.
- **ChaoSec** underestimates **8%** of the estimation.
- **ChaoFlat** underestimates **67%** of the estimation.

Result 1: Subject Programs from Prev. Study

— where the observable space is small —



Accuracy

- MSE(**Empirical**) ≫ MSE(**Miller**), MSE(**ChaoFlat**), MSE(**ChaoSec**)

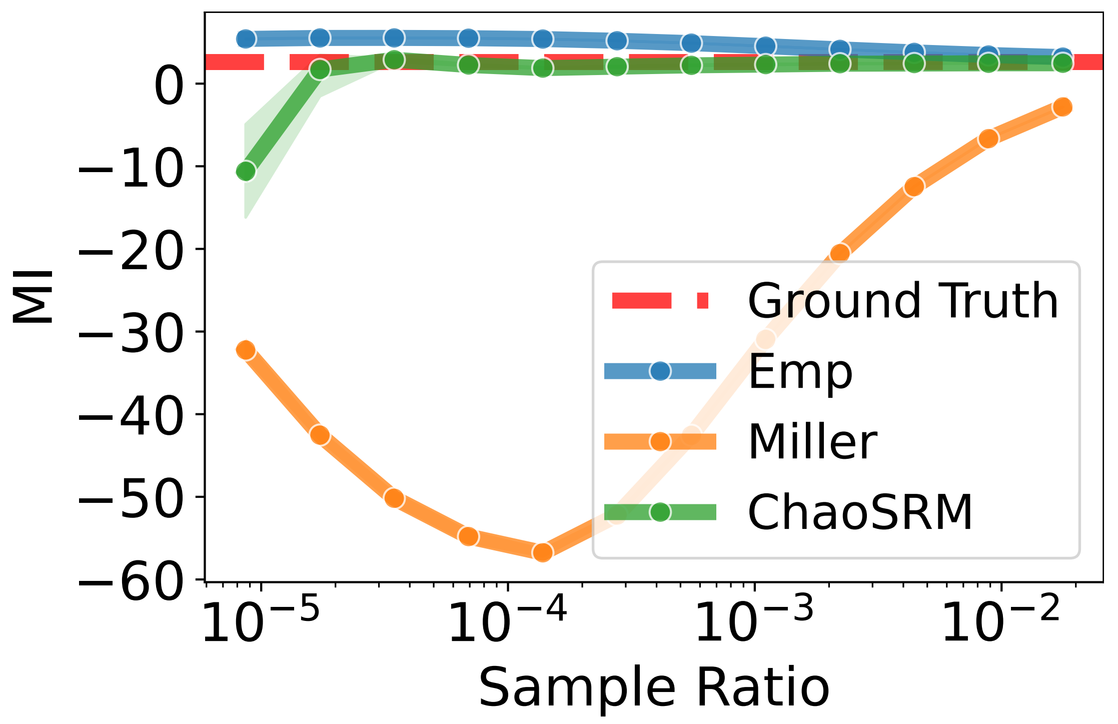
Our **ChaoSec** estimator is the best estimator in terms of both **safety** and **accuracy**.

The **Miller** estimator often *unsafely underestimates the MI* unless the sample size is large.

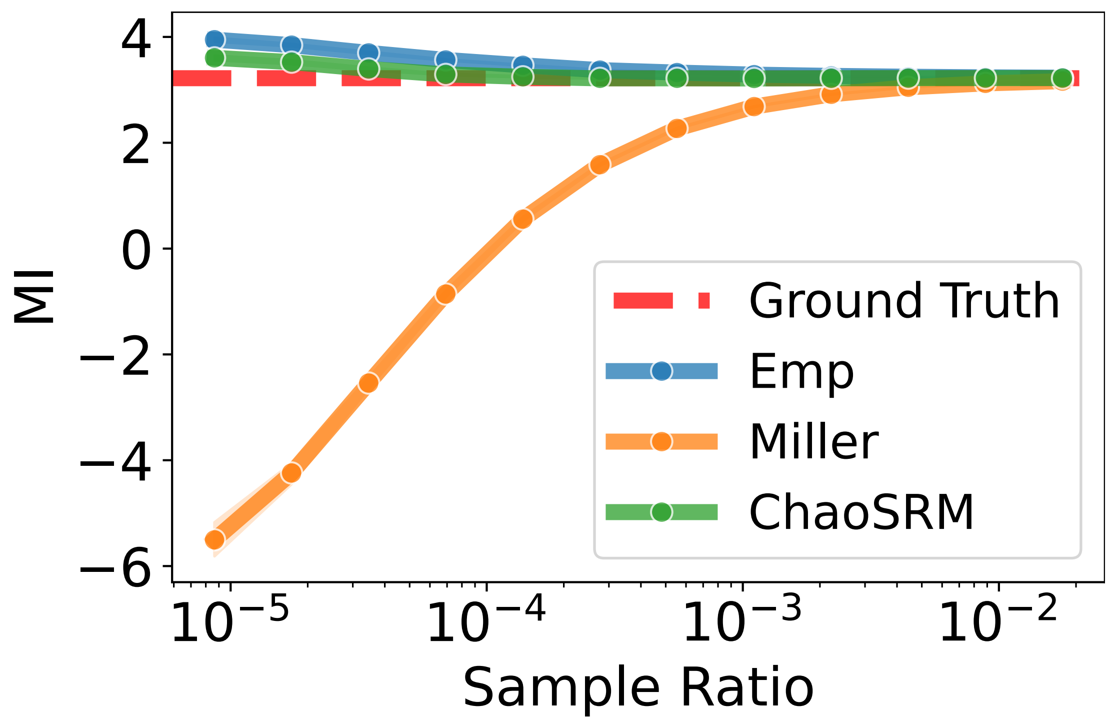
- **Miller** underestimates 57% of the estimation.
- **ChaoSec** underestimates 8% of the estimation.
- **ChaoFlat** underestimates 67% of the estimation.

Result 2: Practical Scenarios

Location Privacy

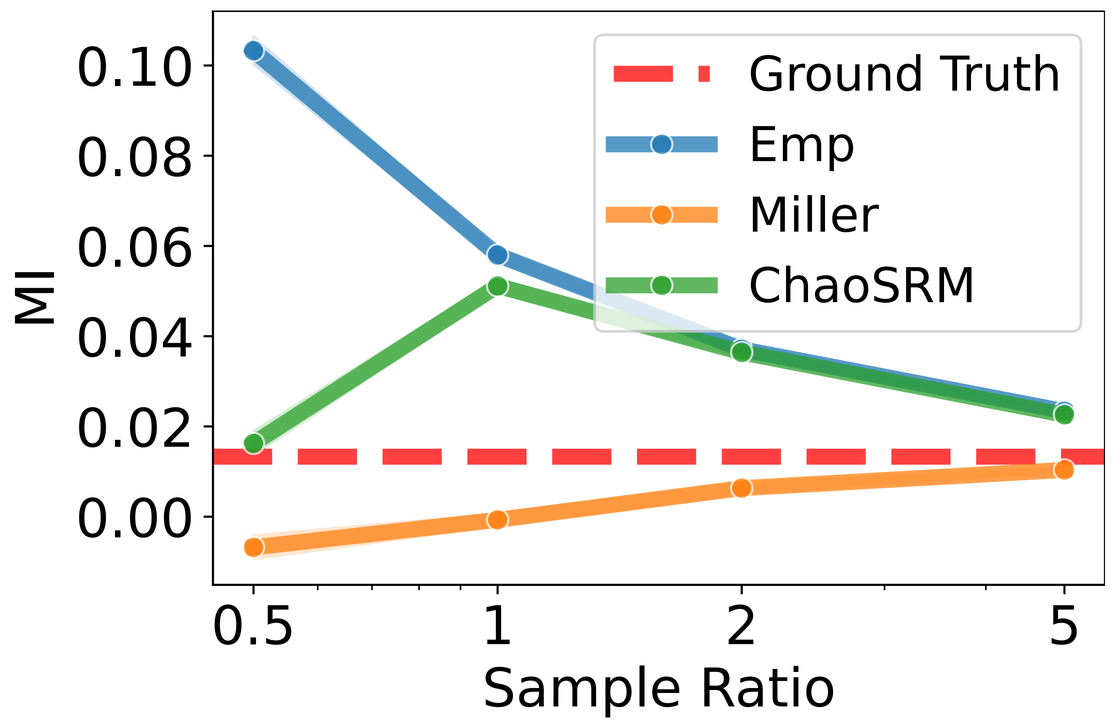


Planner Laplacian

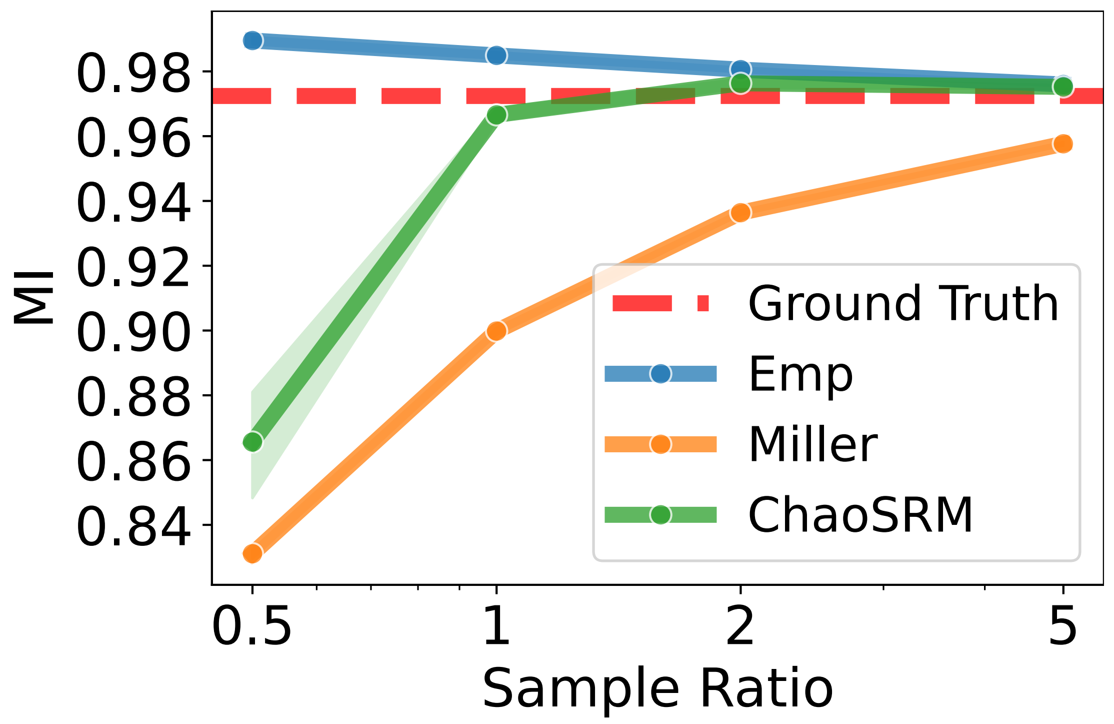


Oya et al.'s LPPM

Passport Tracing



British, Fixed



British, Unfixed

- The ***Domain of the joint event space*** ⟨🗝️,🔍⟩ ***is substantially larger*** than the previous subject programs.
- **Miller** estimator significantly underestimates (even < 0) due to the large bias correction term.

[*Accuracy*] **ChaoSec** $>$ **Empirical** $\gg$ **Miller** [*Safety*] **Empirical** $\approx$ **ChaoSec** $\gg$ **Miller**

Research **Aim**

Empirical estimator

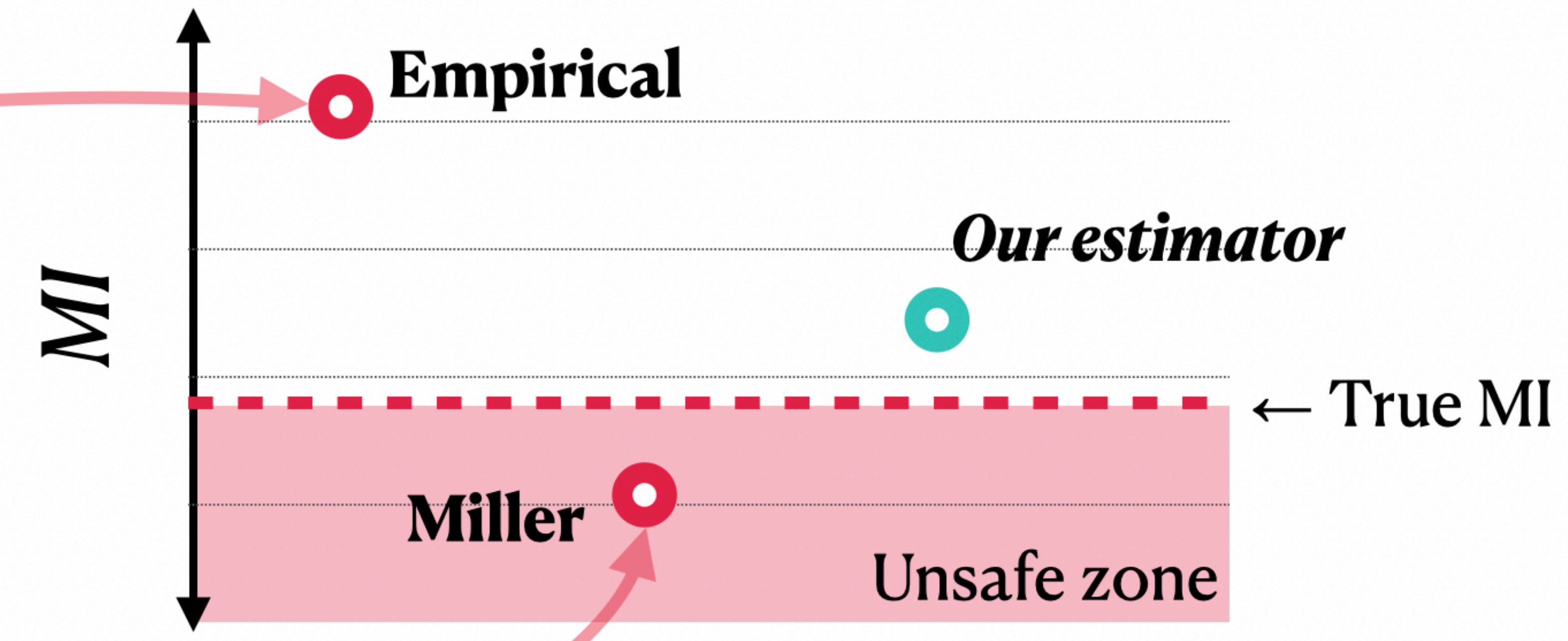
$$\hat{I}_{emp}(S;O) = \hat{H}_{emp}(X) - \hat{H}_{emp}(X \mid Y)$$

Inaccurate

Miller estimator

$$\hat{I}_{miller} = \hat{I}_{emp} - \frac{(m_S - 1)(m_O - 1)}{2n}$$

Unsafe w/ small samples



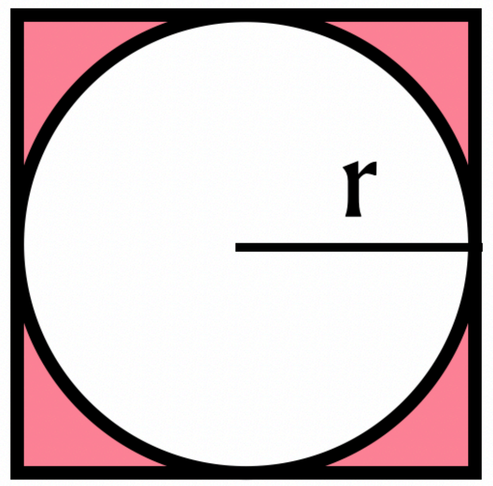
We developed an estimator that ***accurately*** *and* ***safely*** *estimates the mutual information in the presence of* ***missing or rare events***.

How Correct/Secure is our Software?

Q. What is the probability of a thrown 🎾 ball to the 🟥 square dropped not into the ⚪ circle?

1️⃣ Analytic methodology

If the problem can easily be ***mathematically*** *modeled, (e.g, area = circle)*

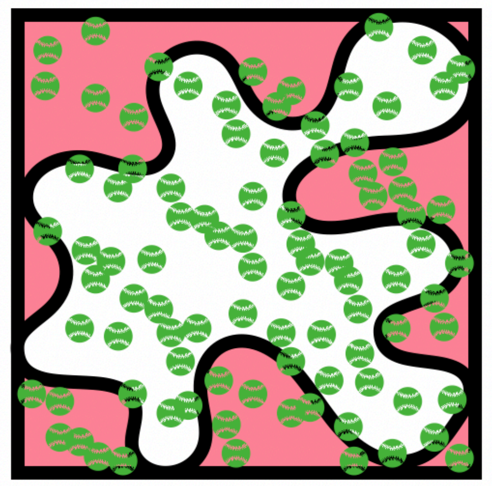


$$\Pr(\neg\text{in circle}) = \frac{Area(\text{Square}) - Area(\text{Circle})}{Area(\text{square})} = \frac{(2r)^2 - \pi r^2}{(2r)^2} = \frac{4-\pi}{4} \approx 0.2146...$$

✅ ***Precise result / Formal guarantees***

2️⃣ Empirical methodology

For example, the ***Monte Carlo method****, where we* ***simulate*** *the ball throwing*

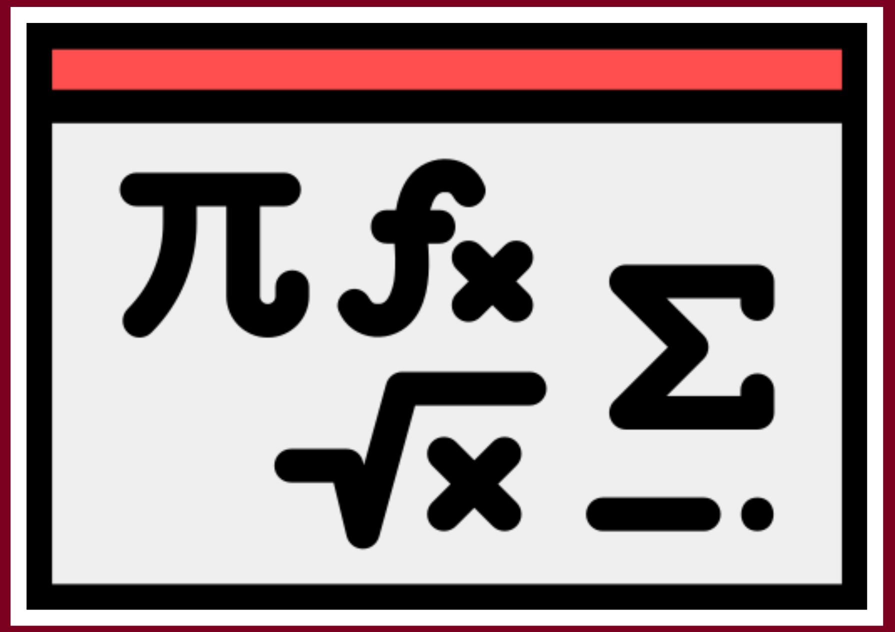


$$\hat{\Pr}(\neg\text{in area}) = \frac{\#\text{ of balls outside the area}}{\#\text{ of balls thrown}} = \frac{3577}{10000} = 0.3577$$

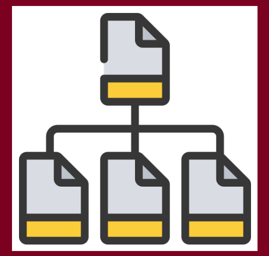
✅ ***Scalable****, i.e., can deal with complex problems*

Two Ways to answer How Correct/Secure is our Software?

Analytical Methods

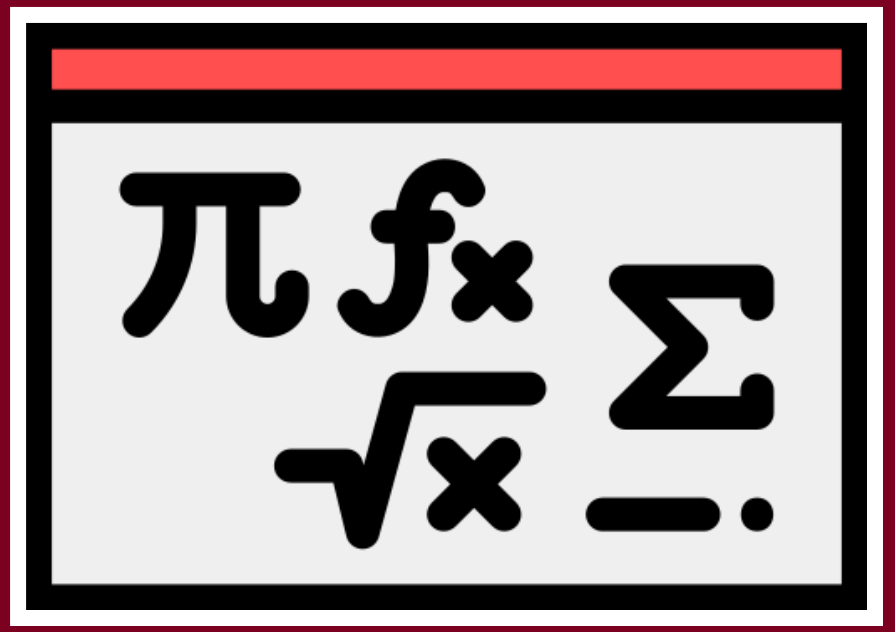


Mathematical proof can provide a ***formal guarantee***



Scalability issues on modern software

Analytical Methods



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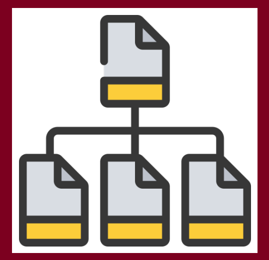
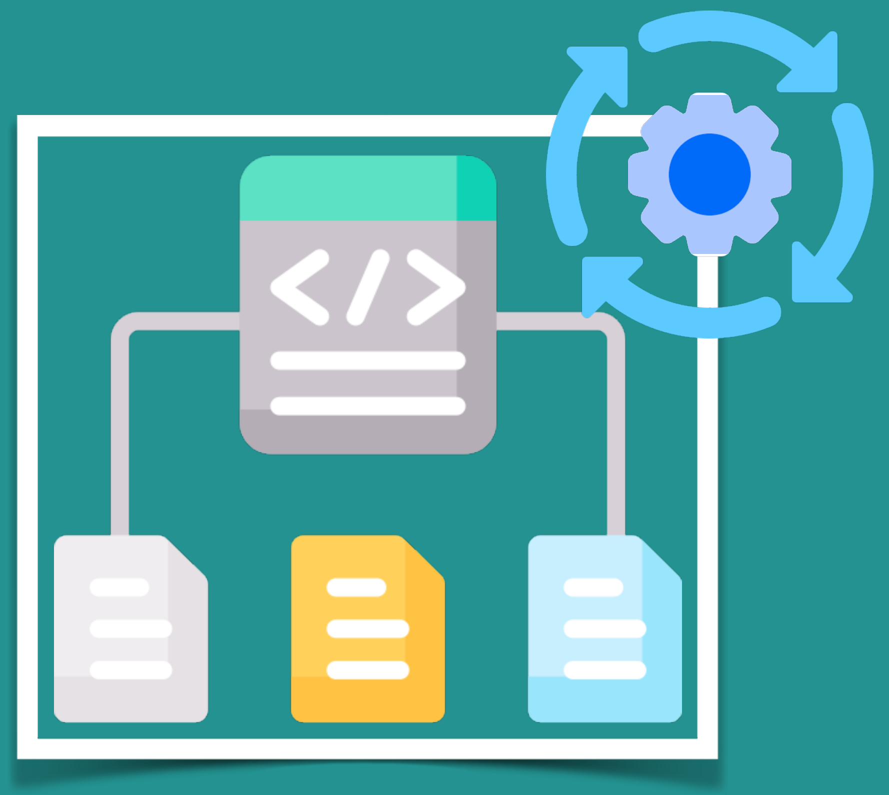


Scalability issues on modern software

Empirical Methods

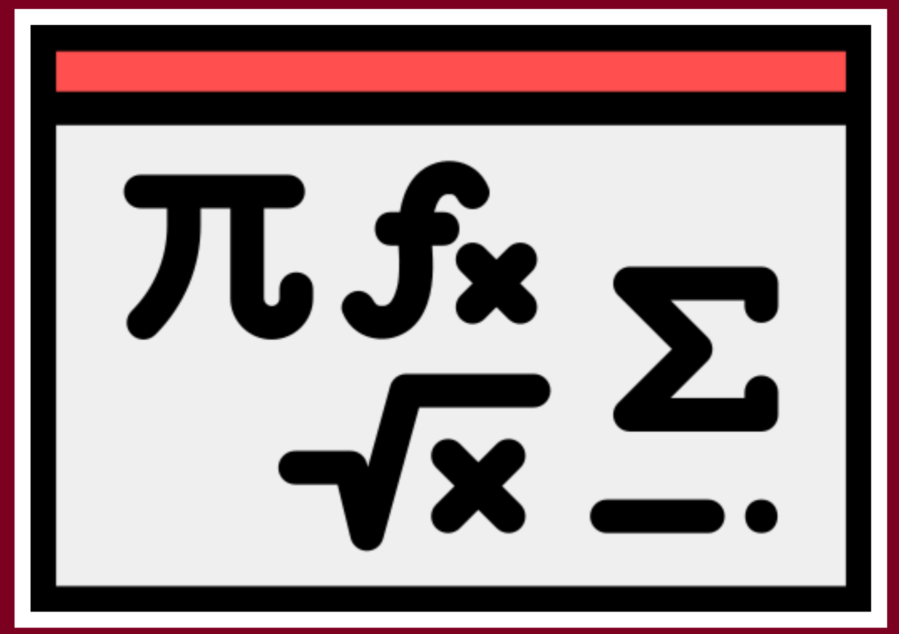


Test software by running it with various test executions

By actually running the software, it solves the ⚠️ scalability issue

There is always **unseen**
⇒ ***No guarantee***

Analytical Methods



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Scalability issues on modern software

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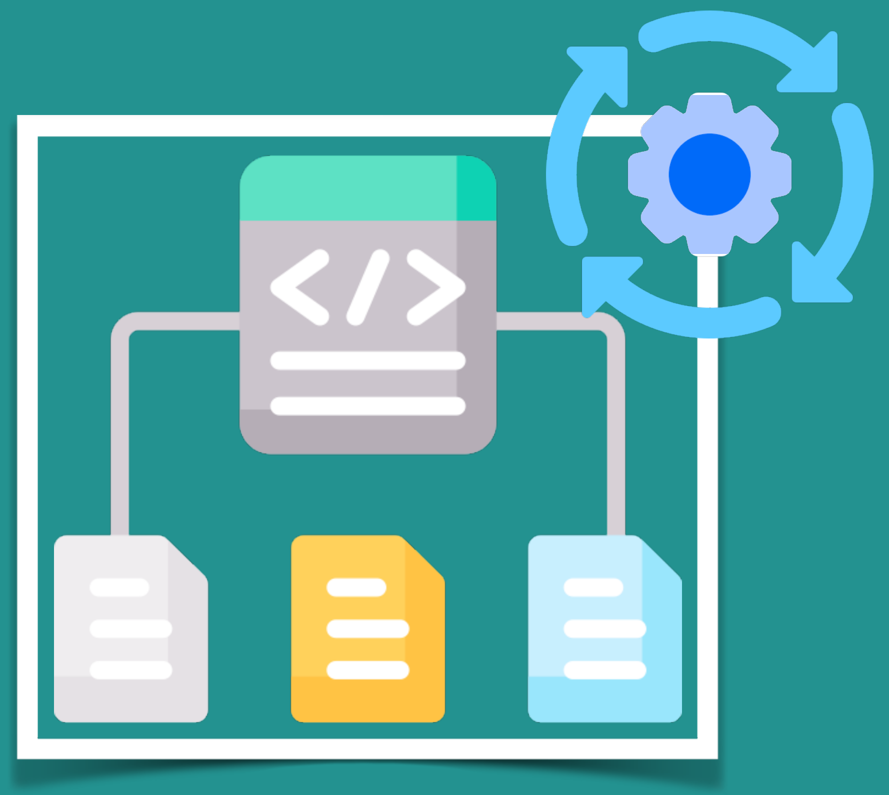

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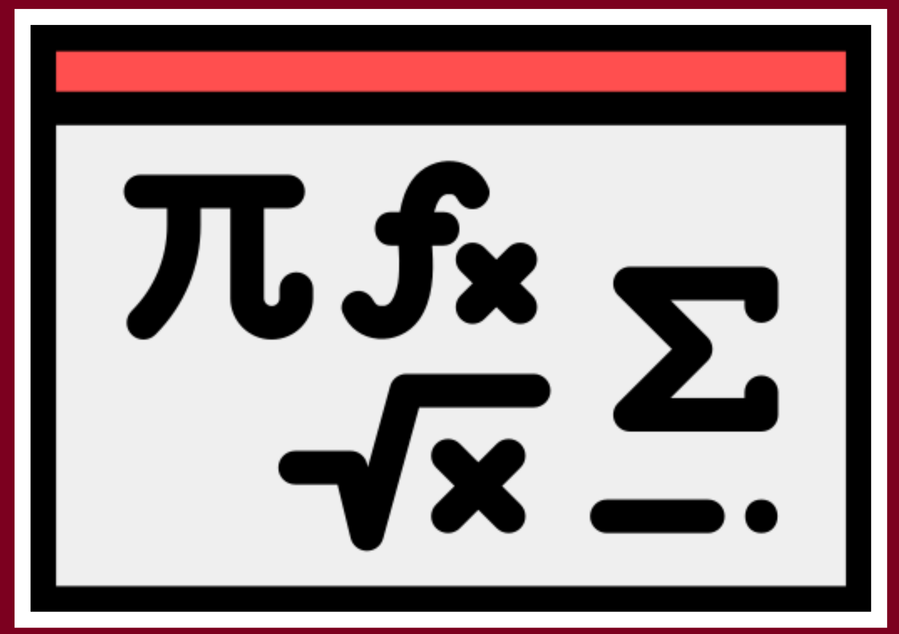
Analytical Methods



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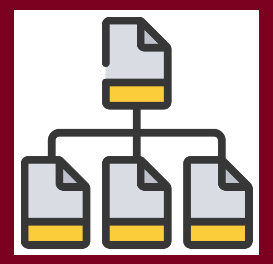
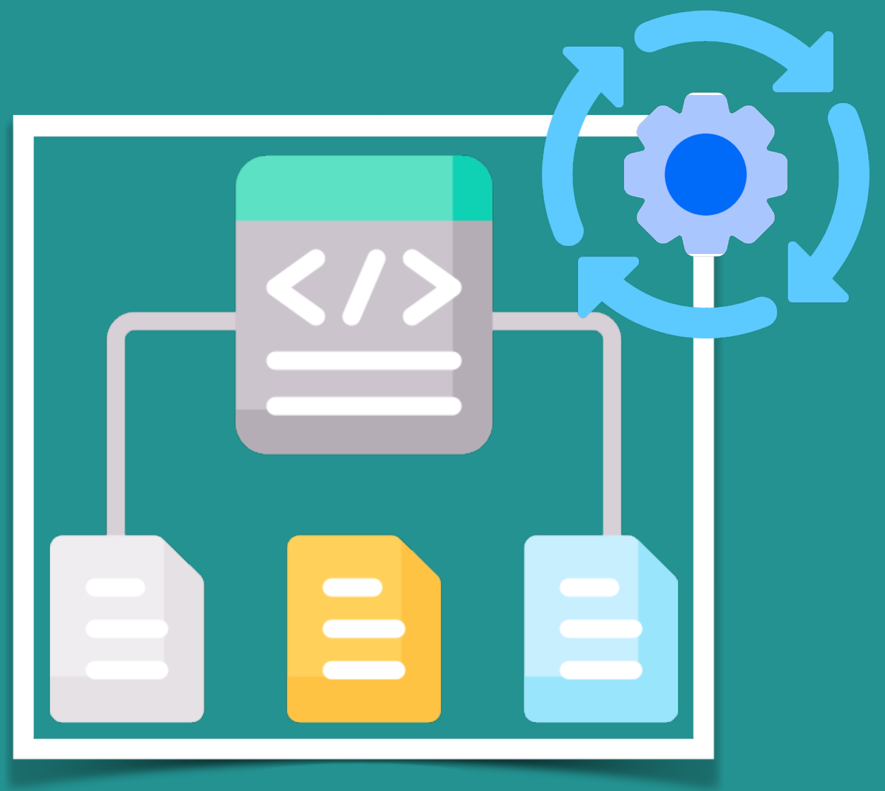


⚠ Scalability issues on modern software

Empirical Methods



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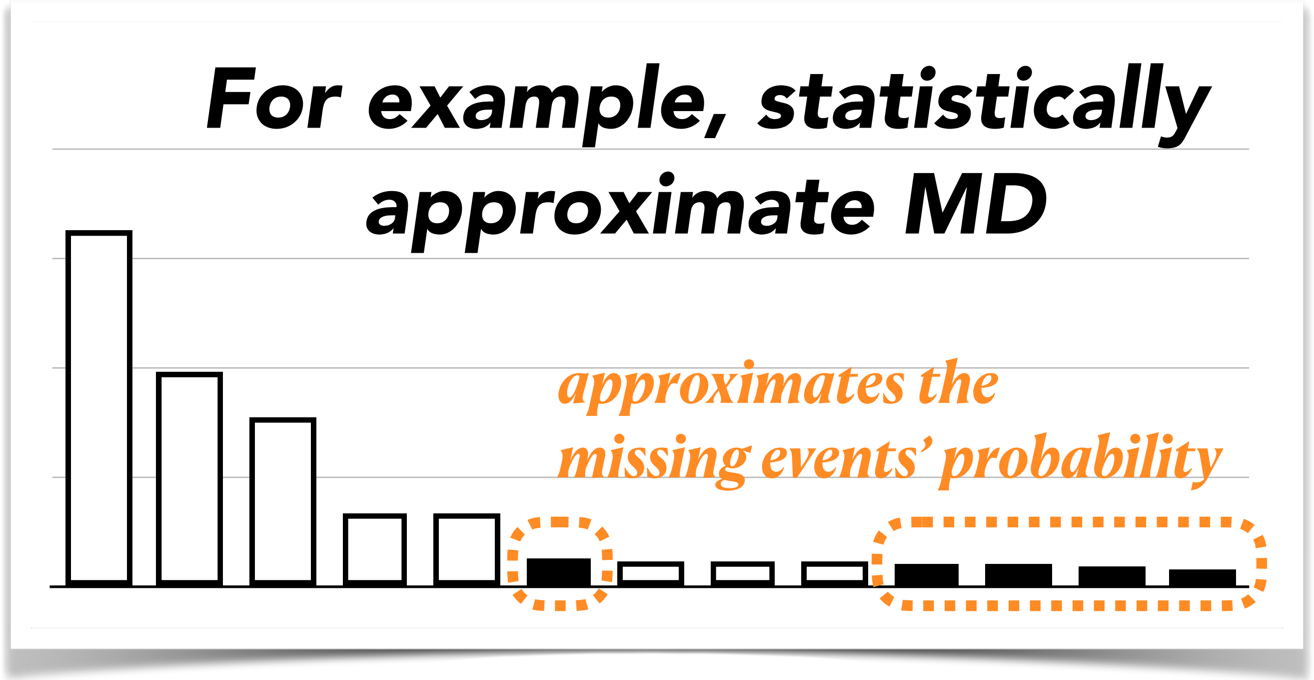

Statistics can solve this!

Accounting for Missing Events in Statistical Information Leakage Analysis

MAX PLANCK INSTITUTE
FOR SECURITY AND PRIVACY

Research Aim

Empirical estimator

$\hat{I}_{emp}(S;O) = \hat{H}_{emp}(X) - \hat{H}_{emp}(X \mid Y)$

Inaccurate

Miller estimator

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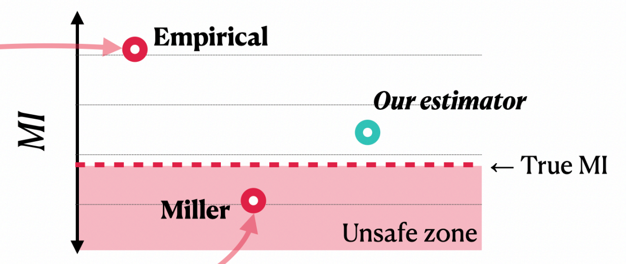

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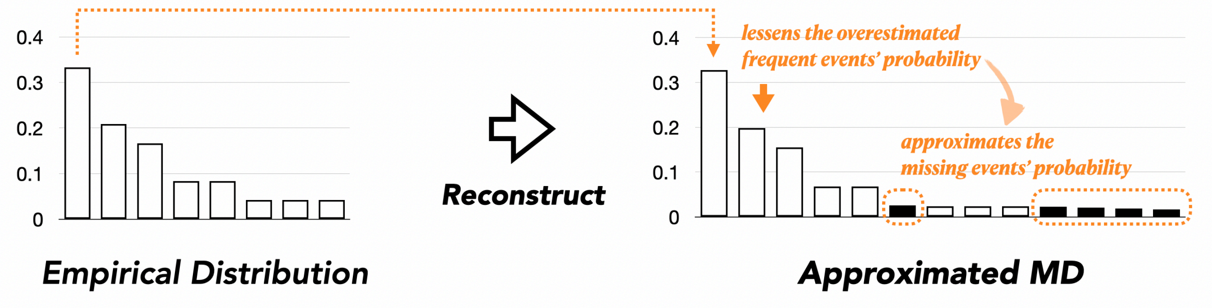
Chao's Multinomial Distribution (MD) Estimation

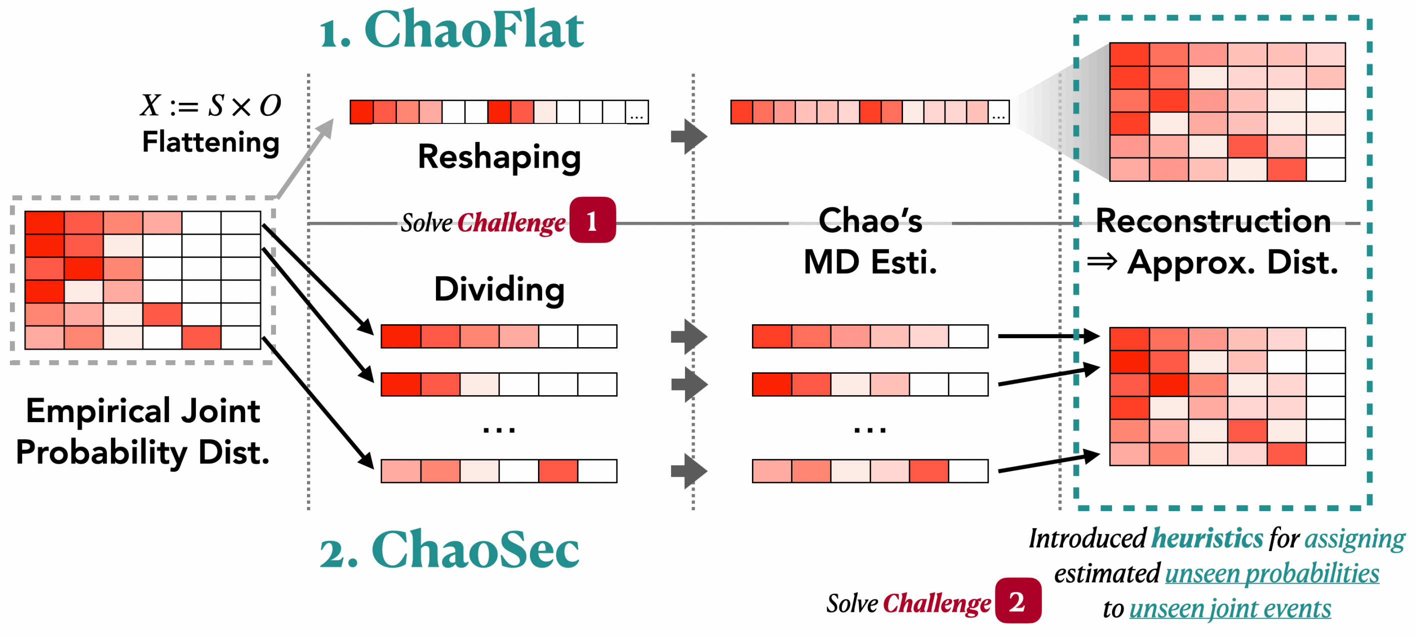
- Given a samples from the unknown multinomial distribution (MD), Chao's MD **reconstruct the underlying MD** by approximation.

➡ *Handle the missing/rare events problem* 😀

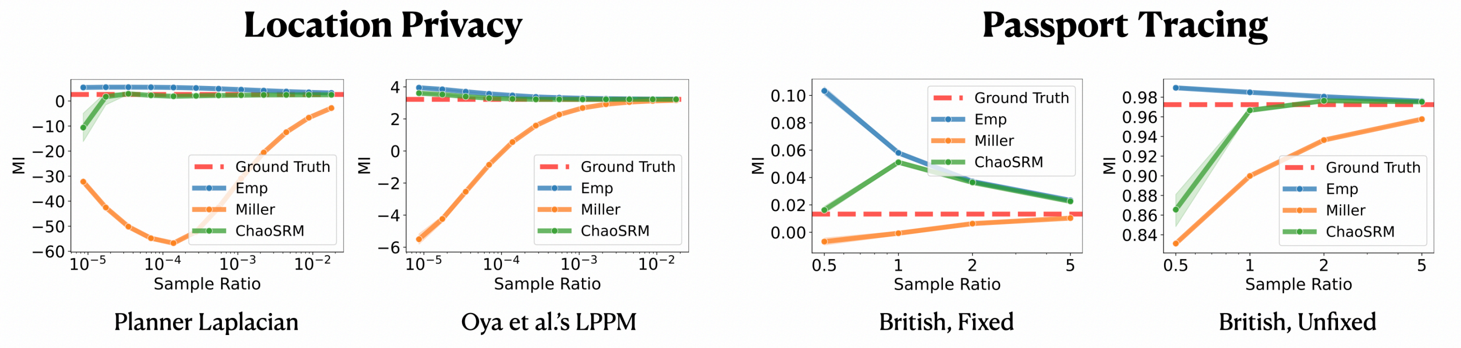


A. Chao et al, "Unveiling the species-rank abundance distribution by generalizing the good-turing sample coverage theory." Ecology, vol. 96 5, pp. 1189–1201, 2015.

Our Approach to estimate MI



Result 2: Practical Scenarios



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[Accuracy] ChaoSec > Empirical ≫ Miller *[Safety]* Empirical ≈ ChaoSec ≫ Miller

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** This work has been done during his internship @ MPI-SP before joining Georgia Tech.*